CSE 152: Computer Vision

Hao Su

Lecture 7: Neural Networks

Review of Filters:
From Linear to Non-linear

Image filtering (Linear case)

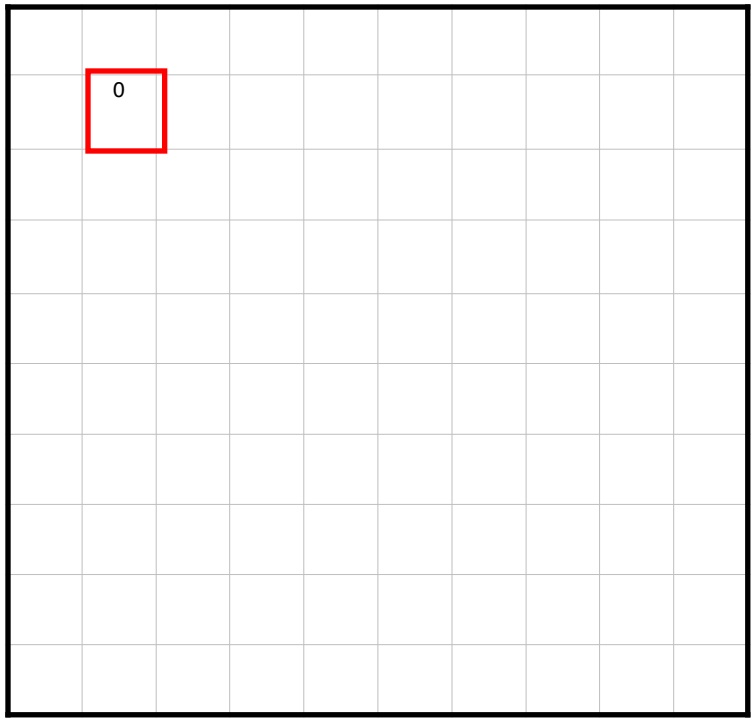
$f[\cdot,\cdot]$ $\frac{1}{9}$

1	1	1
1	1	1
1	1	1

$I[.,.]$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$h[.,.]$



$$h[m,n] = \sum_{k,l} f[k,l]\, I[m+k, n+l]$$

Credit: S. Seitz

Image filtering (Linear case)

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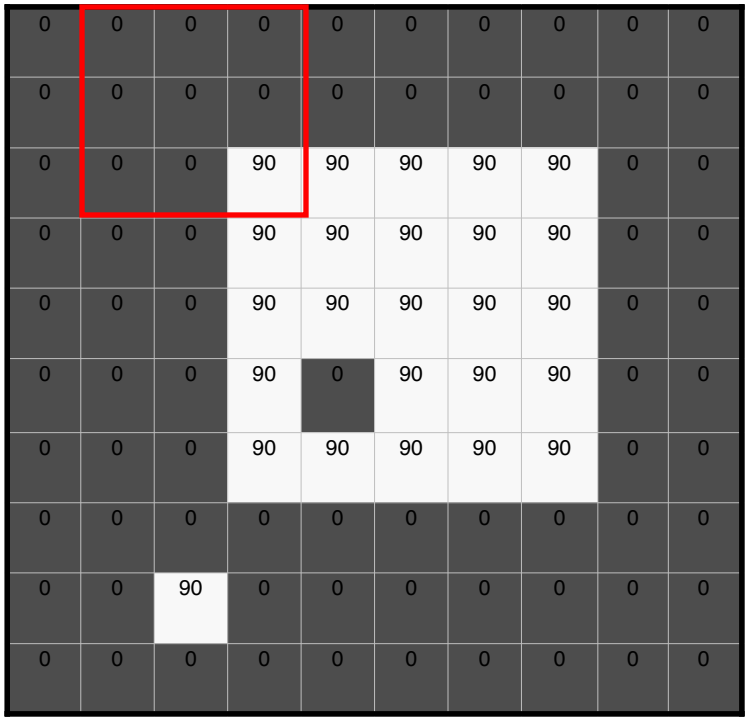

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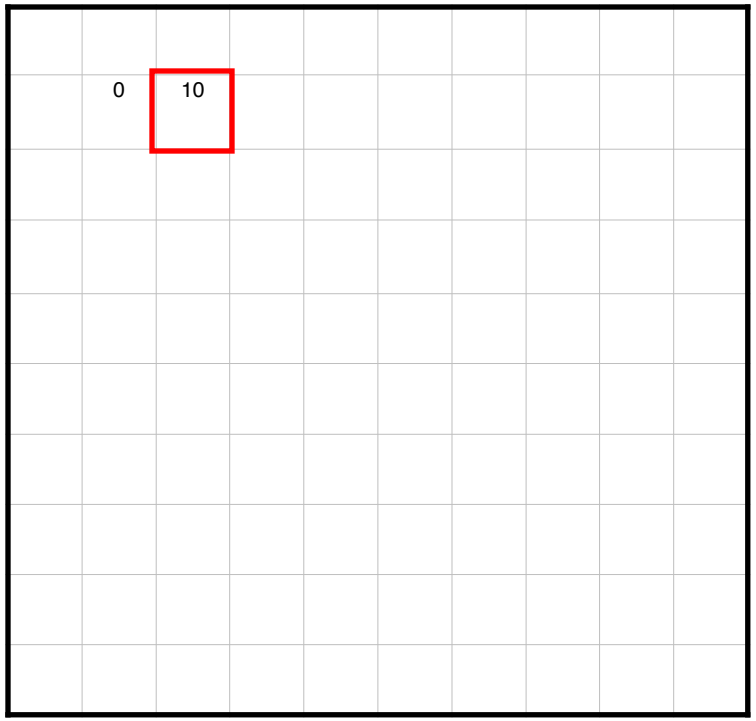

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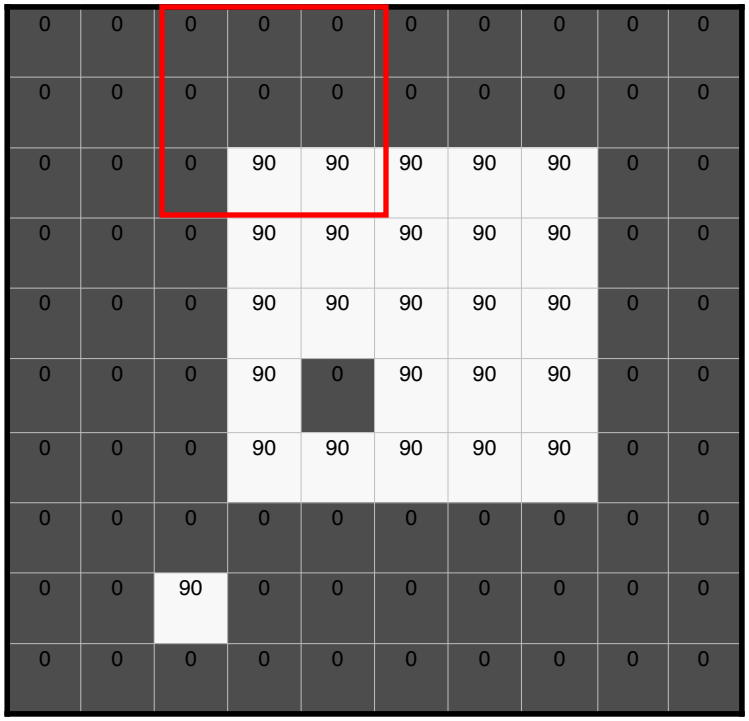
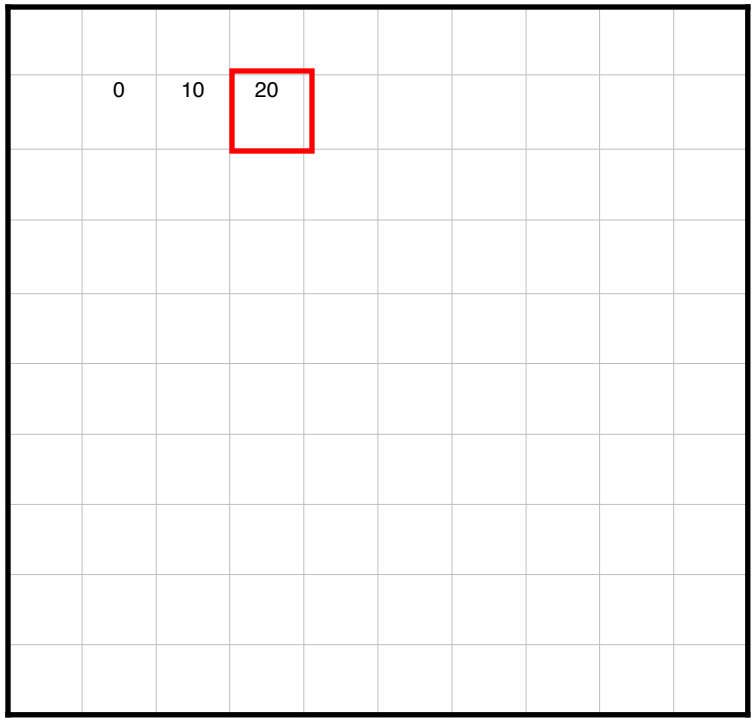


$h[.,.]$



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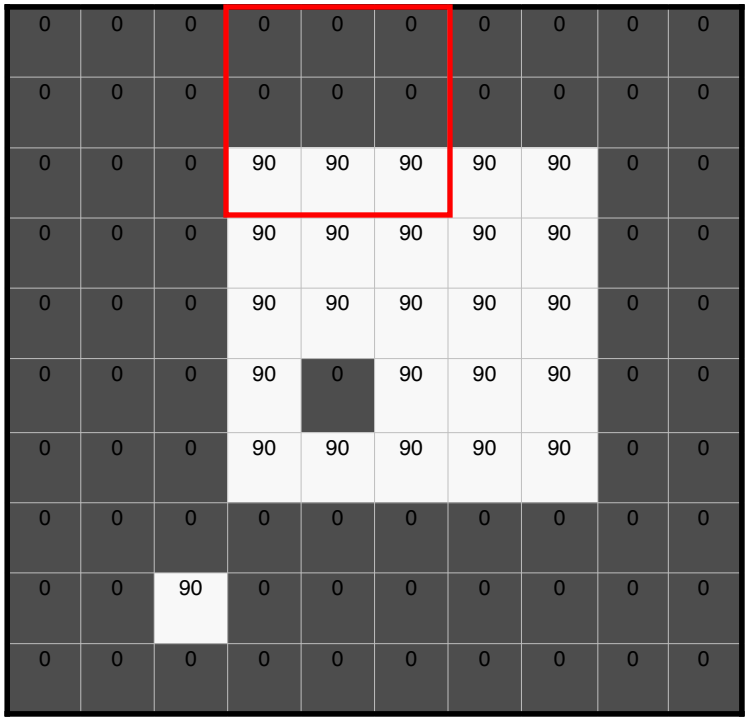

$h[.,.]$

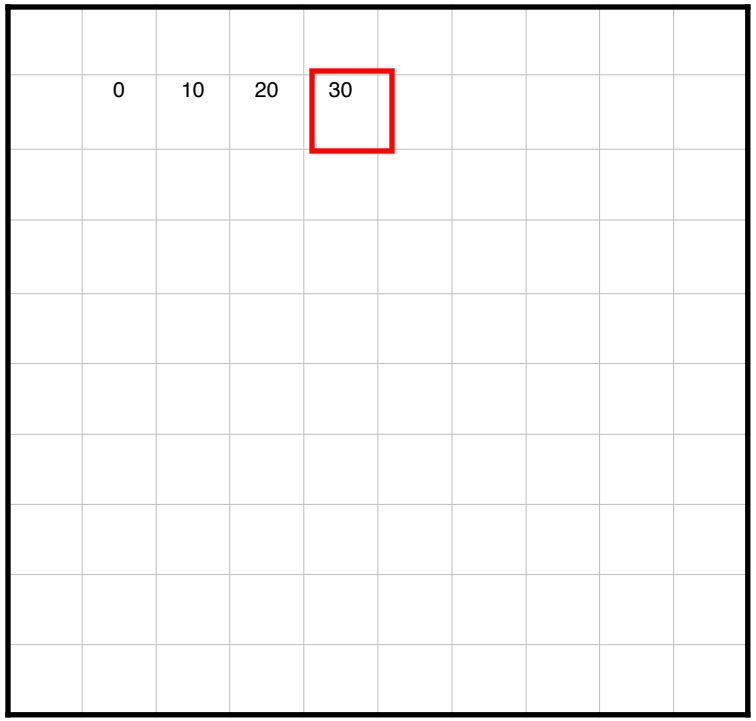

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Image filtering (Linear case)

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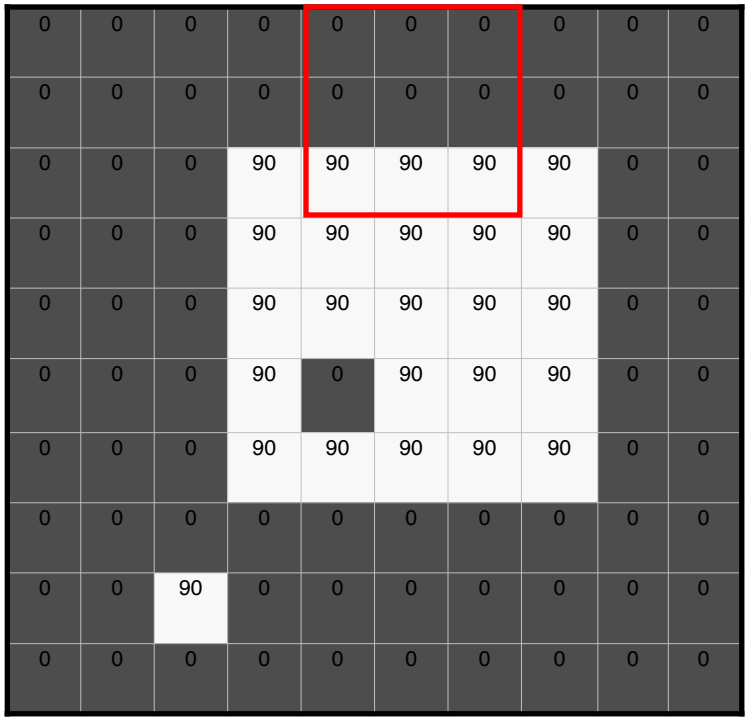
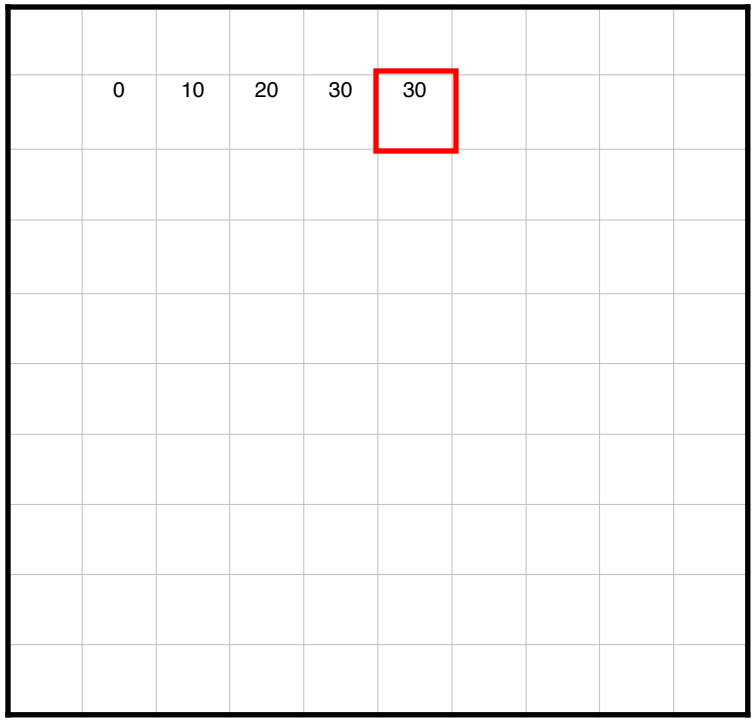


$h[.,.]$



$$h[m,n] = \sum_{k,l} f[k,l]\, I[m+k, n+l]$$

Credit: S. Seitz

Image filtering (Linear case)

$f[\cdot,\cdot]$ $\frac{1}{9}$

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$I[.,.]$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$h[.,.]$

0	10	20	30	30	30	20	10
0	20	40	60	60	60	40	20
0	30	60	90	90	90	60	30
0	30	50	80	80	90	60	30
0	30	50	80	80	90	60	30
0	20	30	50	50	60	40	20
10	20	30	30	30	30	20	10
10	10	10	0	0	0	0	0

$$h[m,n] = \sum_{k,l} f[k,l]\, I[m+k, n+l]$$

Credit: S. Seitz

Reducing salt-and-pepper noise

3x3

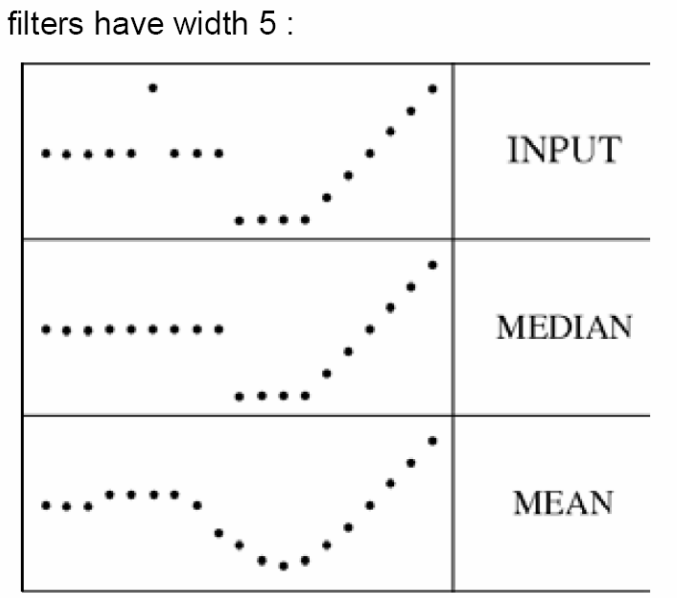
5x5

7x7

- What's wrong with the results?

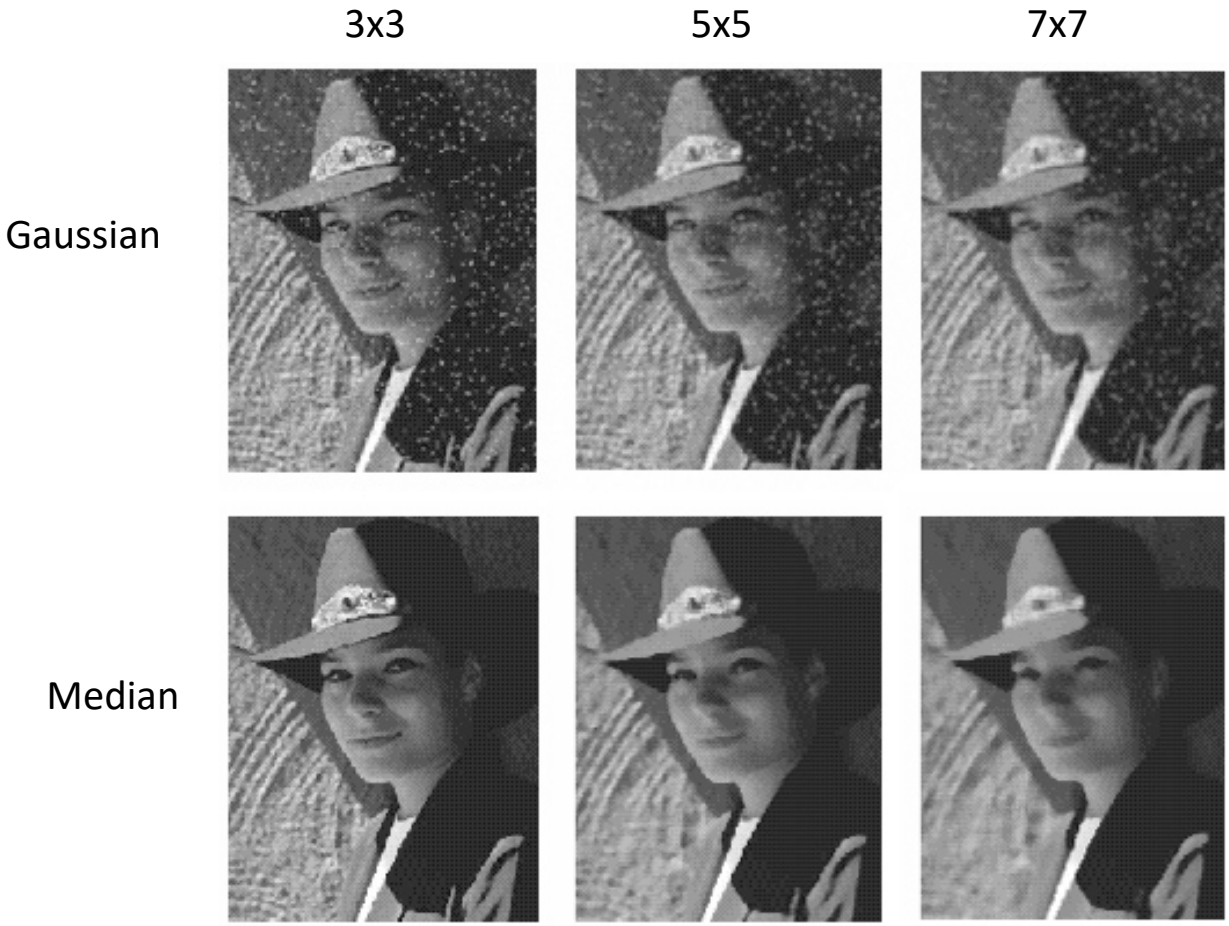
Median filter (Non-linear)

- What advantage does median filtering have over box filtering?
  - Robustness to outliers



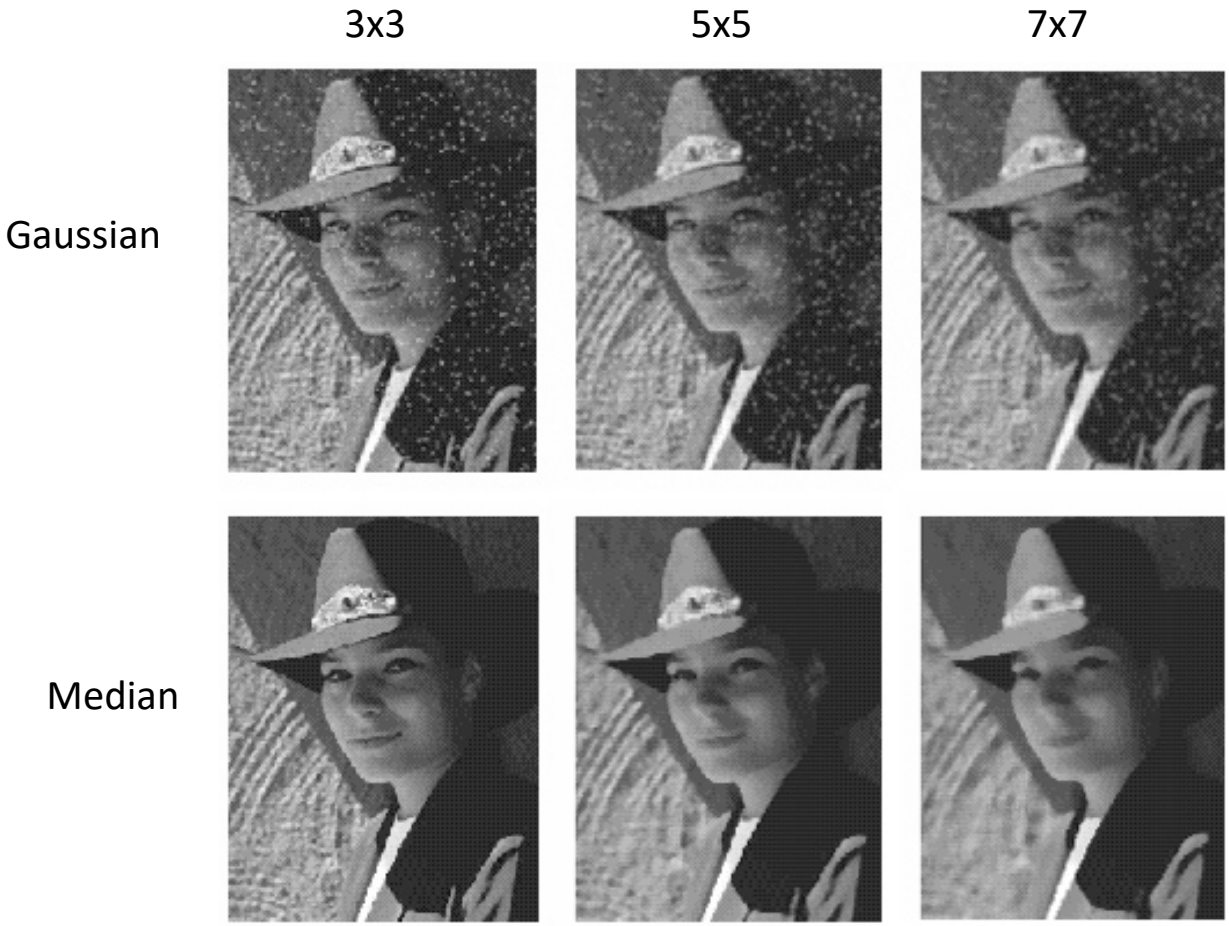
Source: K. Grauman

Median filter (Non-linear)



Source: M. Hebert

Gaussian vs. median filtering



Neural Networks

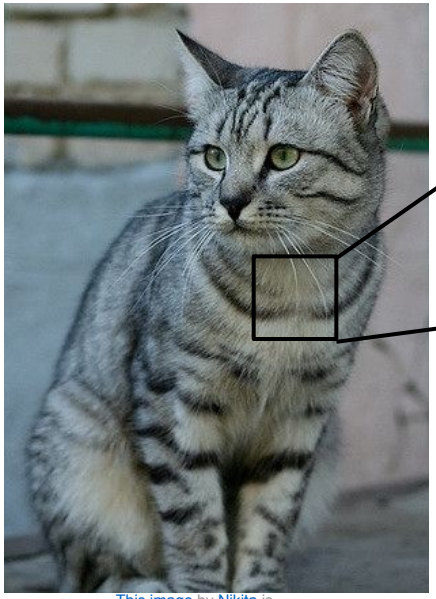
A General Framework
from Linear to Non-linear

Image Classification: A core task in Computer Vision

(assume given set of discrete labels)
{dog, cat, truck, plane, ...}

cat

The Problem: Semantic Gap

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 [ 91  98 102 106 104  79  98 103  99 105 123 136 110 105  94  85]
 [ 76  85  90 105 128 105  87  96  95  99 115 112 106 103  99  85]
 [ 99  81  81  93 120 131 127 100  95  98 102  99  96  93 101  94]
 [106  91  61  64  69  91  88  85 101 107 109  98  75  84  96  95]
 [114 108  85  55  55  69  64  54  64  87 112 129  98  74  84  91]
 [133 137 147 103  65  81  80  65  52  54  74  84 102  93  85  82]
 [128 137 144 140 109  95  86  70  62  65  63  63  60  73  86 101]
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 [ 63  77  86  81  77  79 102 123 117 115 117 125 125 130 115  87]
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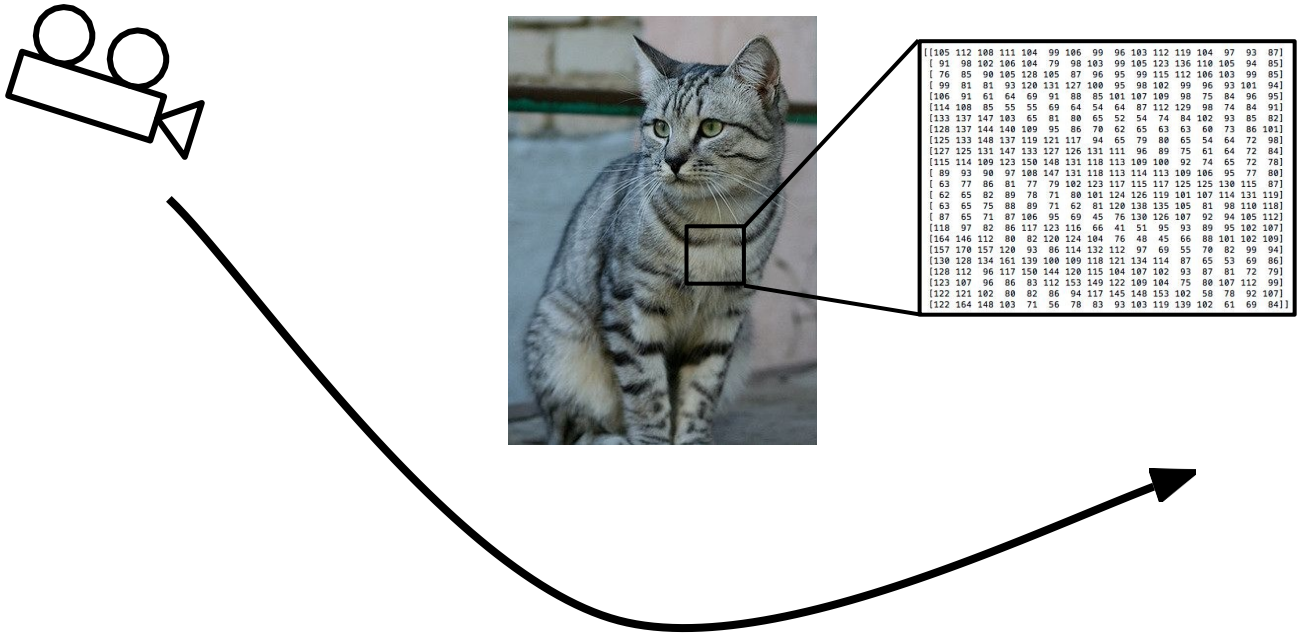
 [123 107  96  86  83 112 153 149 122 109 104  75  80 107 112  99]
 [122 121 102  80  82  86  94 117 145 148 153 102  58  78  92 107]
 [122 164 148 103  71  56  78  83  93 103 119 139 102  61  69  84]]
```

What the computer sees

An image is just a big grid of numbers between [0, 255]:

e.g. 800 x 600 x 3
(3 channels RGB)

Challenges: Viewpoint variation



All pixels change when the camera moves!

Challenges: Illumination

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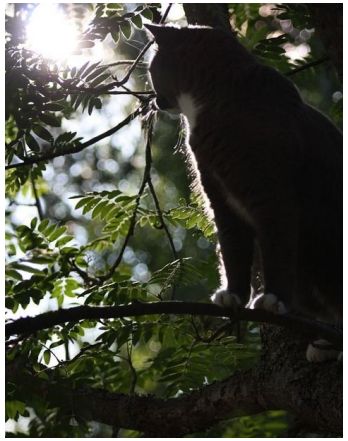

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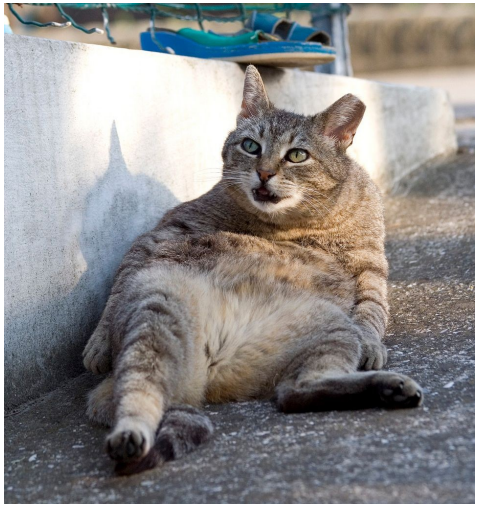
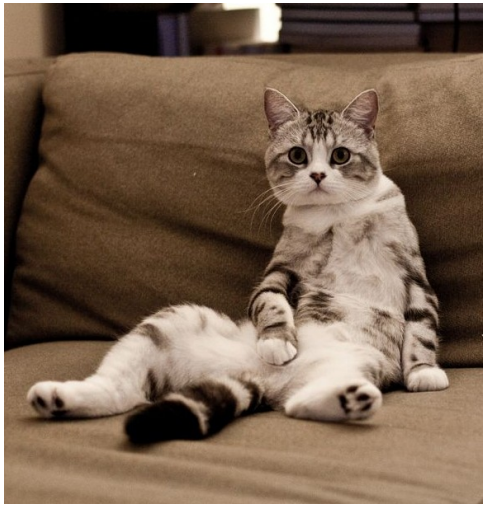
Challenges: Deformation





Challenges: Occlusion

Challenges: Background Clutter

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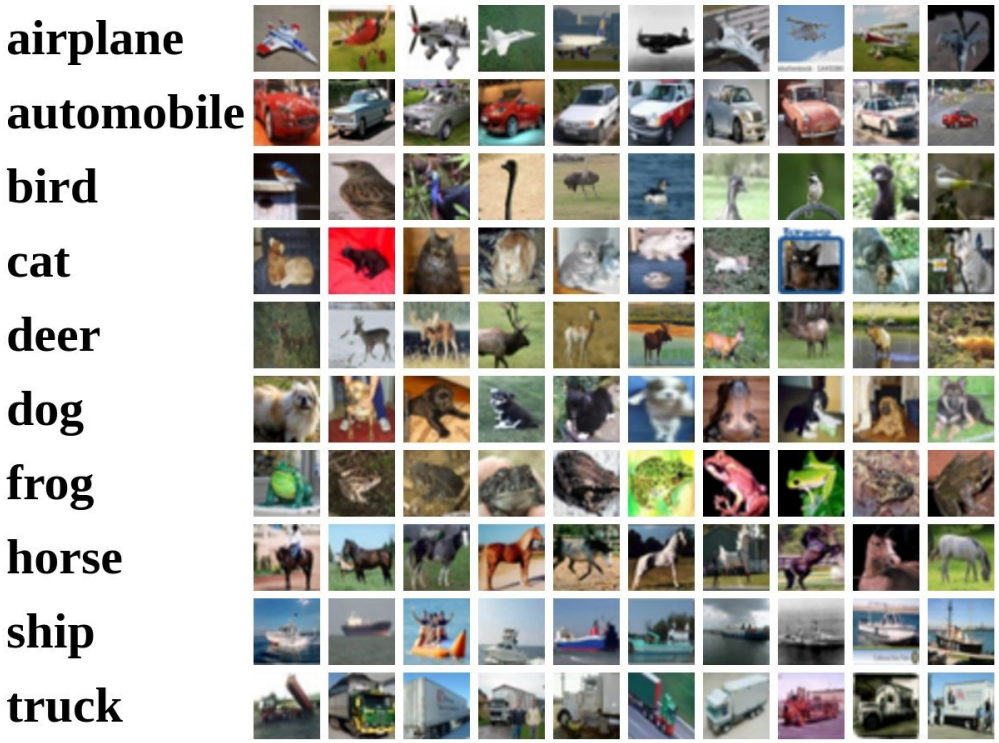
Challenges: Intraclass variation

Linear Classification

Recall CIFAR10



50,000 training images
each image is **32x32x3**

10,000 test images.

Parametric Approach

Image

Array of **32x32x3** numbers
(3072 numbers total)

f(**x**,**W**)

10 numbers giving
class scores

W

parameters
or weights

Parametric Approach: Linear Classifier

$$f(x,W) = Wx$$

Image

Array of **32x32x3** numbers
(3072 numbers total)

f(**x**,**W**)

10 numbers giving
class scores

W

parameters
or weights

Parametric Approach: Linear Classifier

$$f(x,W) = Wx$$

3072x1

10x1

10x3072

Image

Array of **32x32x3** numbers
(3072 numbers total)

f(**x**,**W**)

10 numbers giving class scores

W

parameters or weights

Parametric Approach: Linear Classifier

$$f(x, W) = Wx + b$$

3072x1 — 10x1 — 10x1 — 10x3072

Image

Array of **32x32x3** numbers
(3072 numbers total)

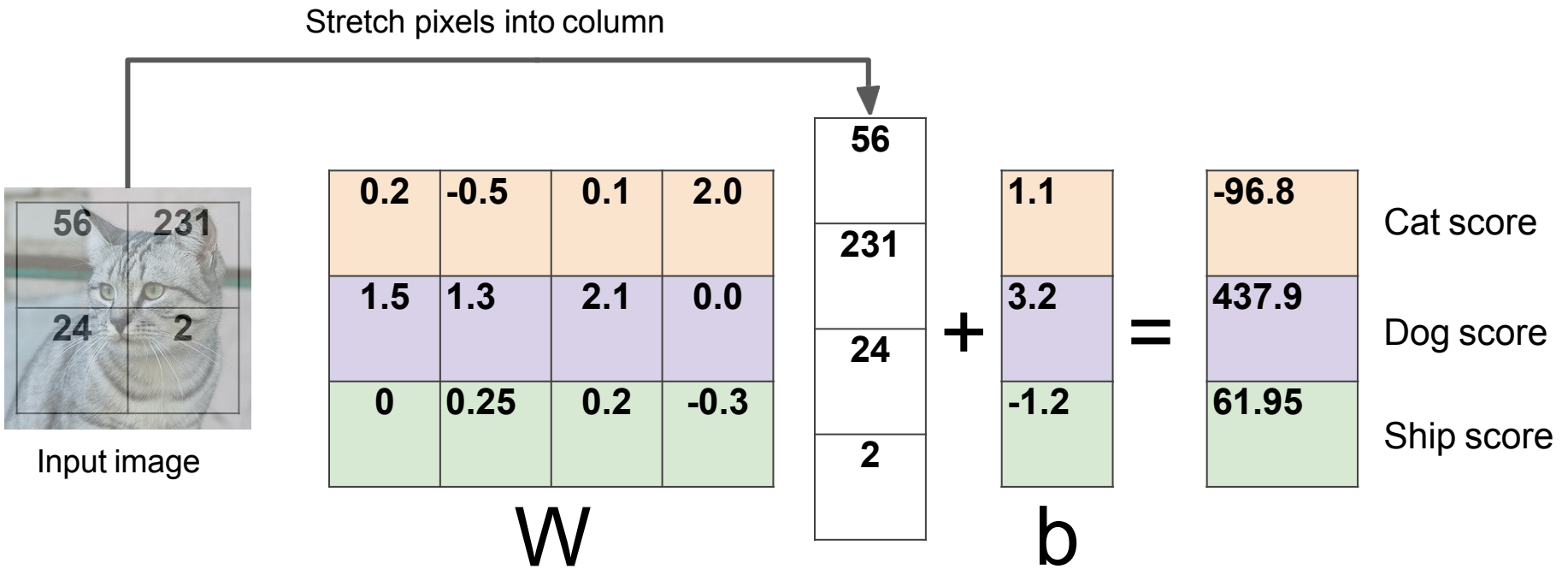
f(**x**,**W**)

10 numbers giving class scores

W

parameters or weights

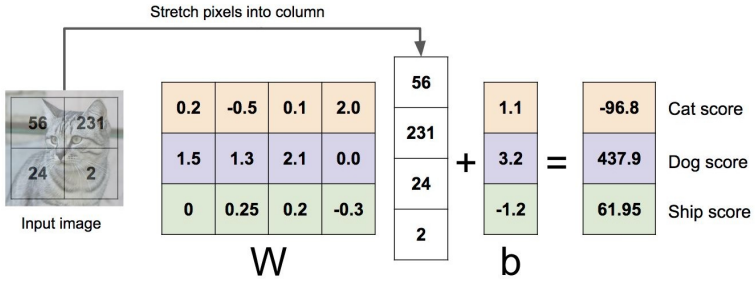
Example with an image with 4 pixels, and 3 classes (cat/dog/ship)



Example with an image with 4 pixels, and 3 classes (cat/dog/ship)

Algebraic Viewpoint

$$f(x,W) = Wx$$



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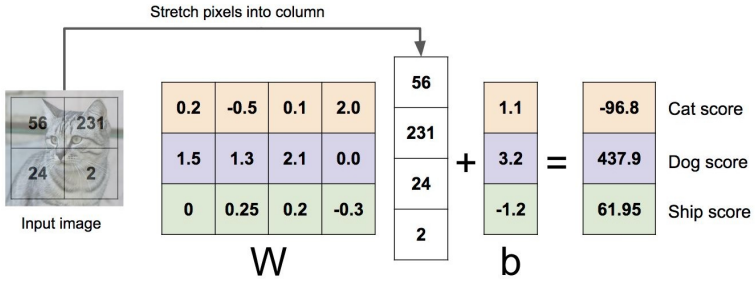
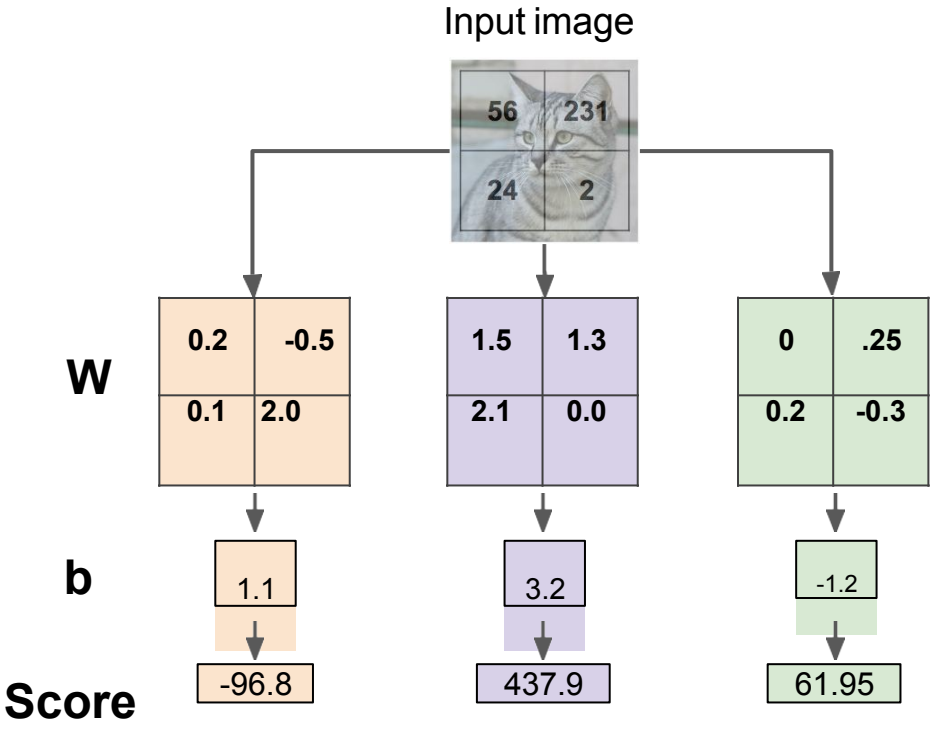
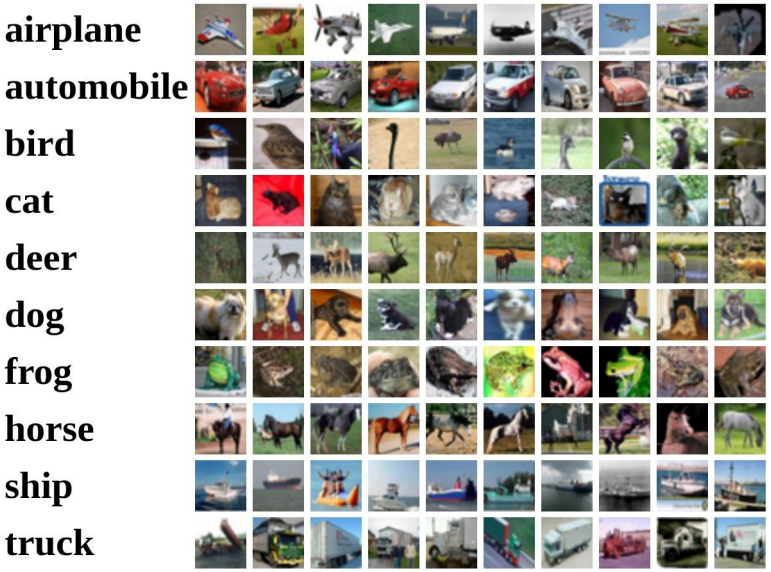
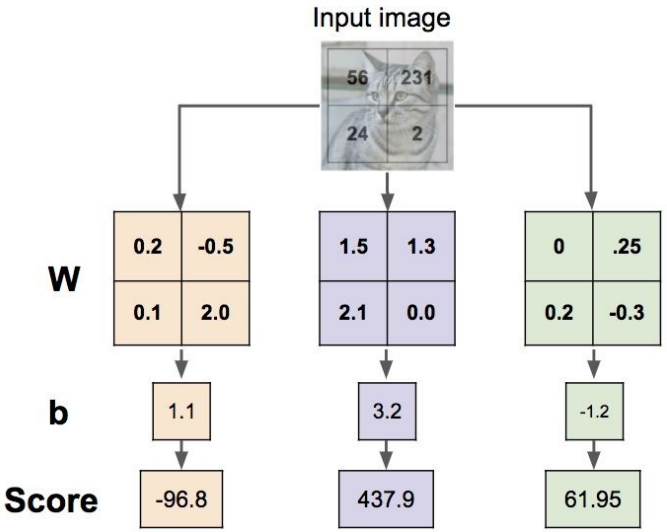


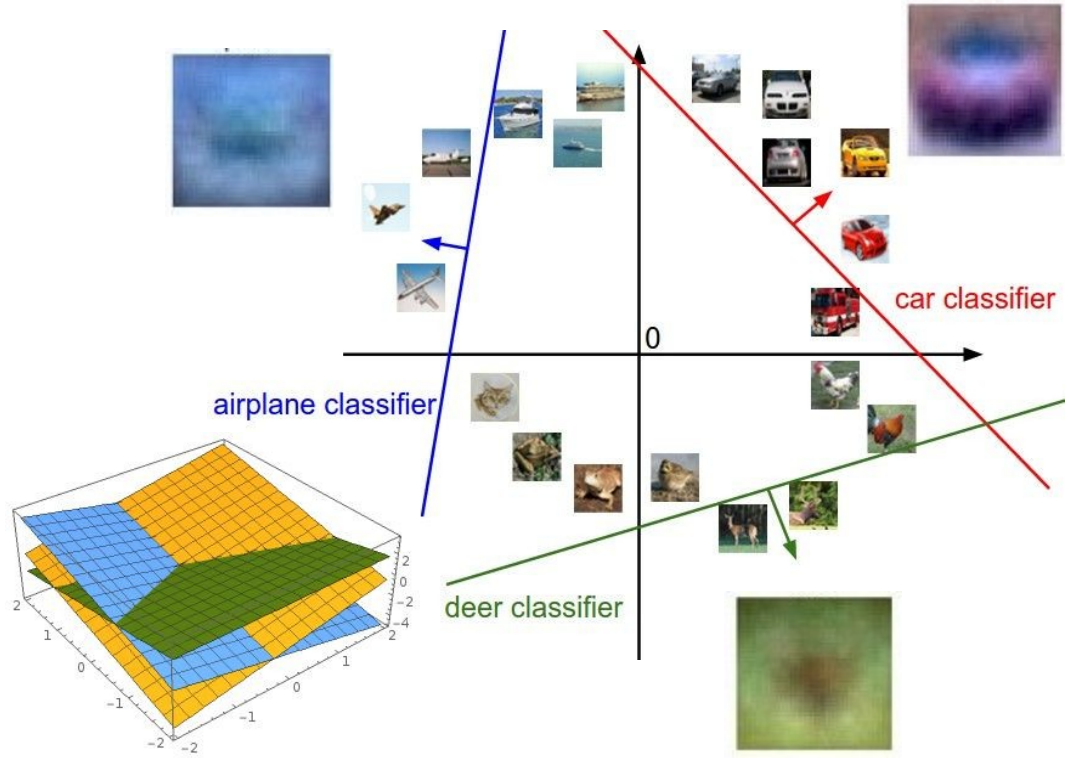


Interpreting a Linear Classifier





Interpreting a Linear Classifier: Geometric Viewpoint



$$f(x,W) = Wx + b$$

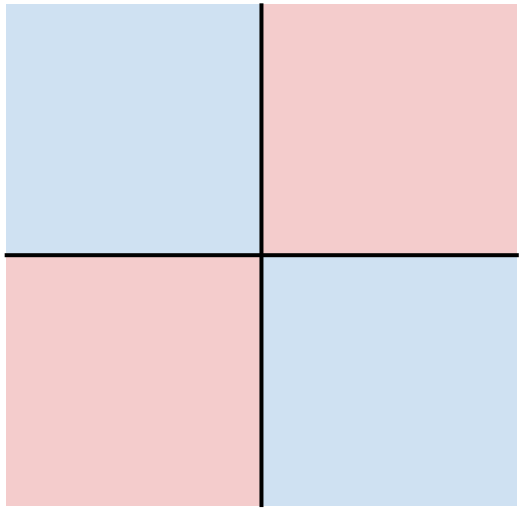
Array of **32x32x3** numbers
(3072 numbers total)

Plot created using Wolfram Cloud

Hard cases for a linear classifier

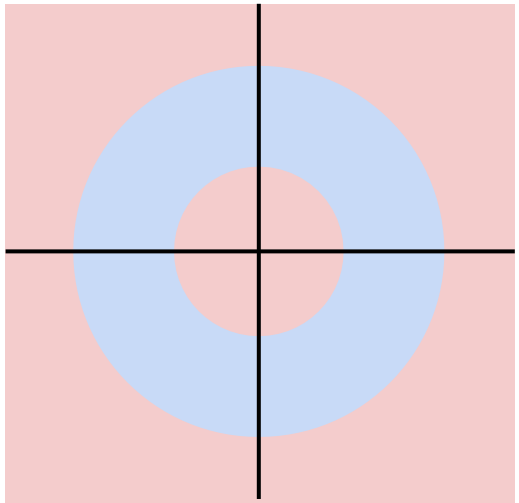
Class 1:
First and third quadrants

Class 2:
Second and fourth quadrants



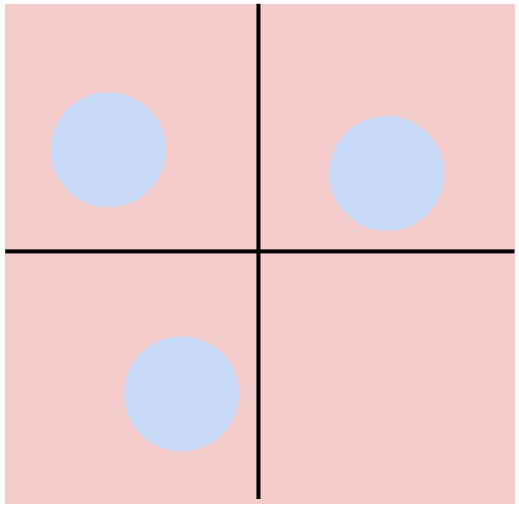
Class 1:
1 <= L2 norm <= 2

Class 2:
Everything else



Class 1:
Three modes

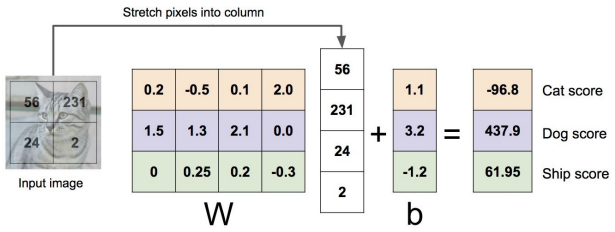
Class 2:
Everything else



Linear Classifier: Three Viewpoints

Algebraic Viewpoint

$$f(x,W) = Wx$$

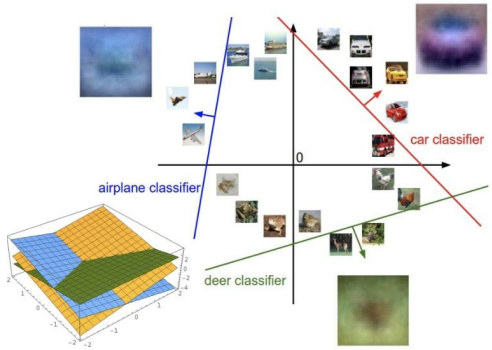


Visual Viewpoint

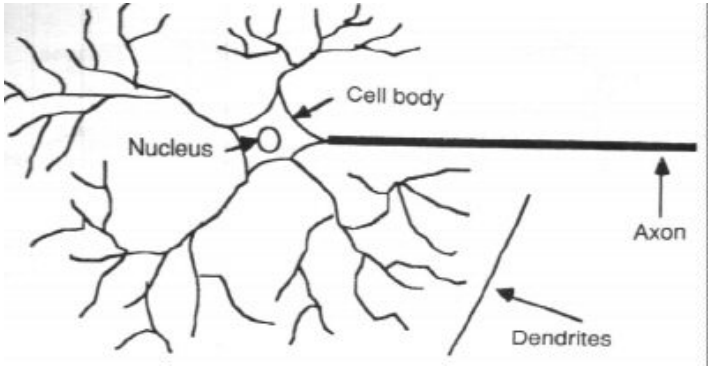
One template per class

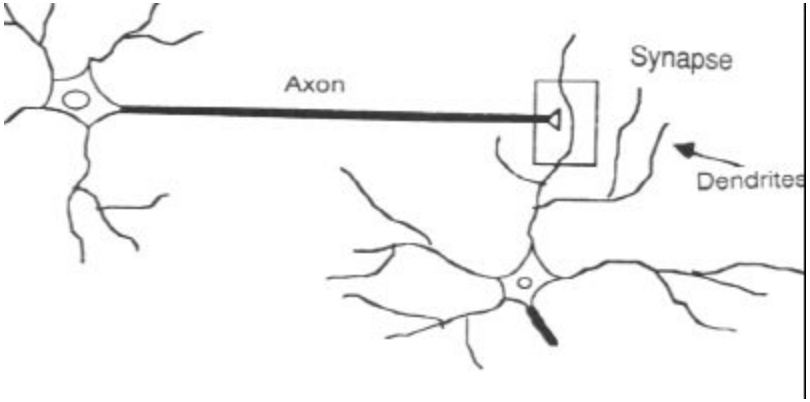
Geometric Viewpoint

Hyperplanes cutting up space



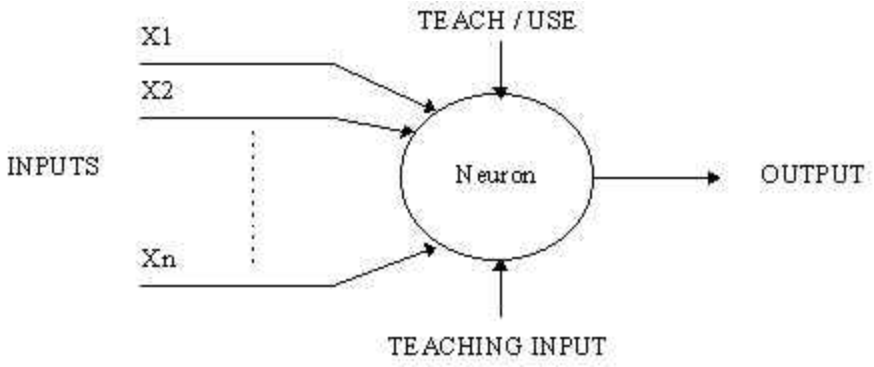
How the Human Brain learns





- In the human brain, a typical neuron collects signals from others through a host of fine structures called *dendrites*.
- The neuron sends out spikes of electrical activity through a long, thin strand known as an *axon*, which splits into thousands of branches.
- At the end of each branch, a structure called a *synapse* converts the activity from the axon into electrical effects that inhibit or excite activity in the connected neurons.

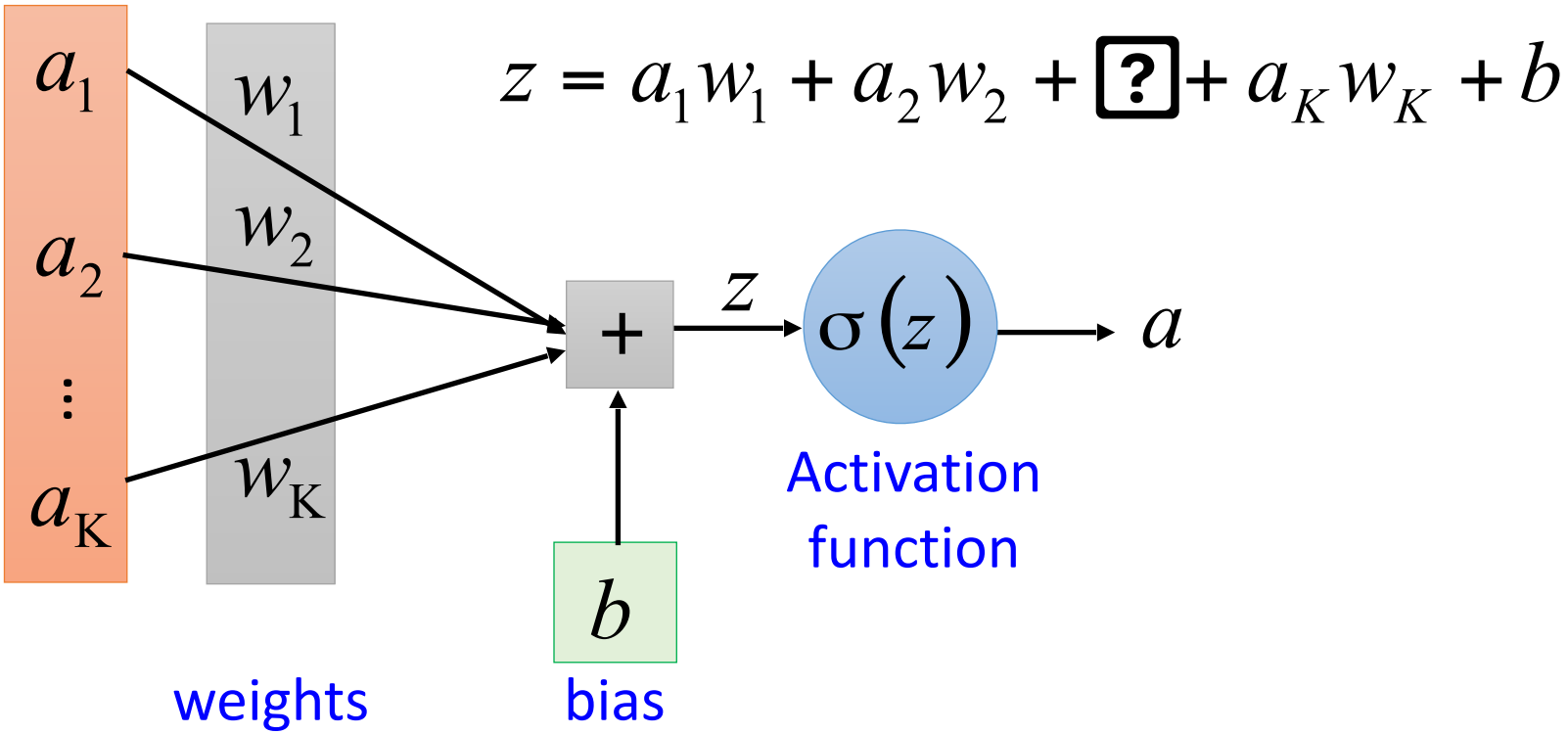
A Simple Neuron



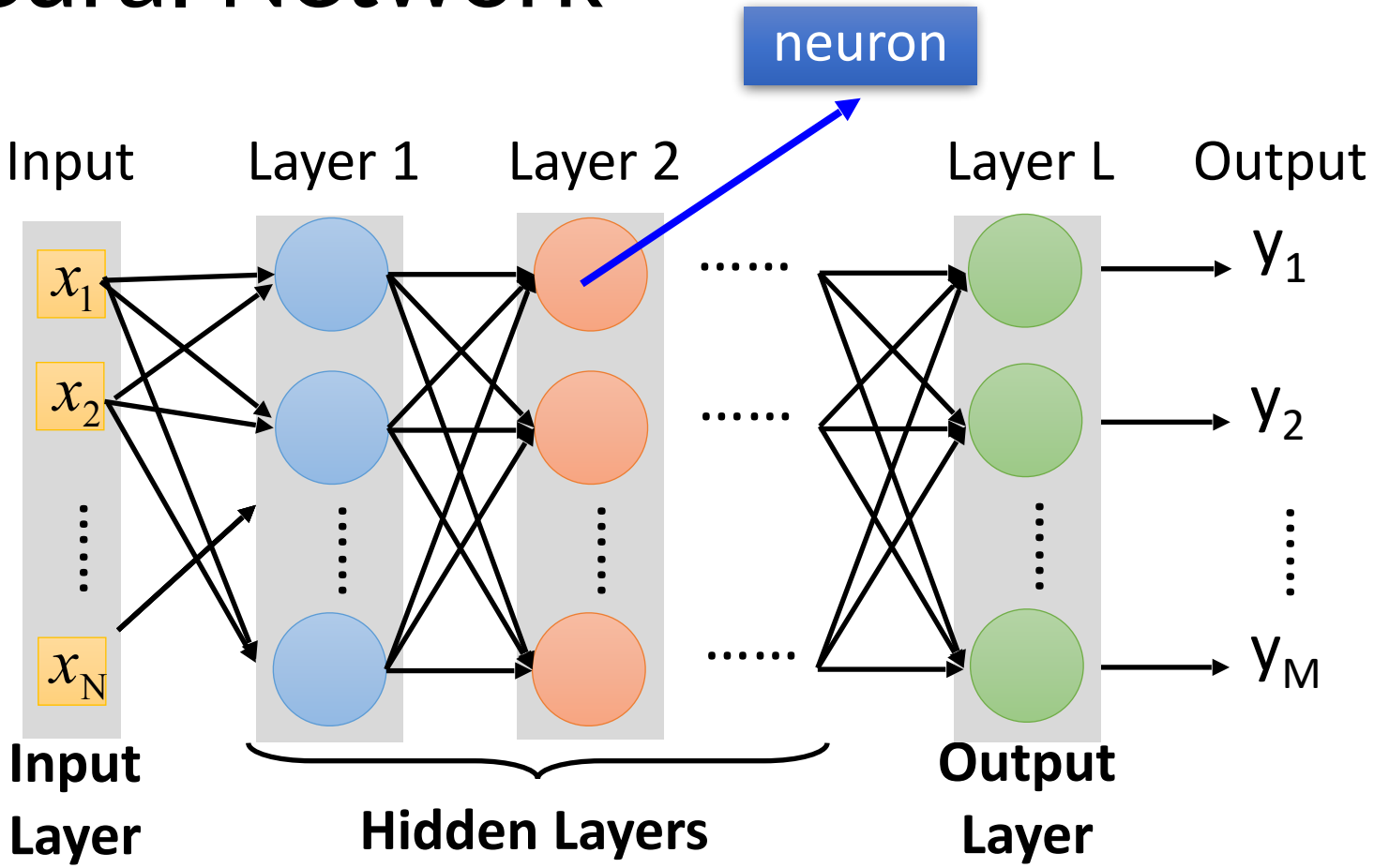
- An artificial neuron is a device with many inputs and one output.

Element of Neural Network

Neuron $f: R^K \rightarrow R$

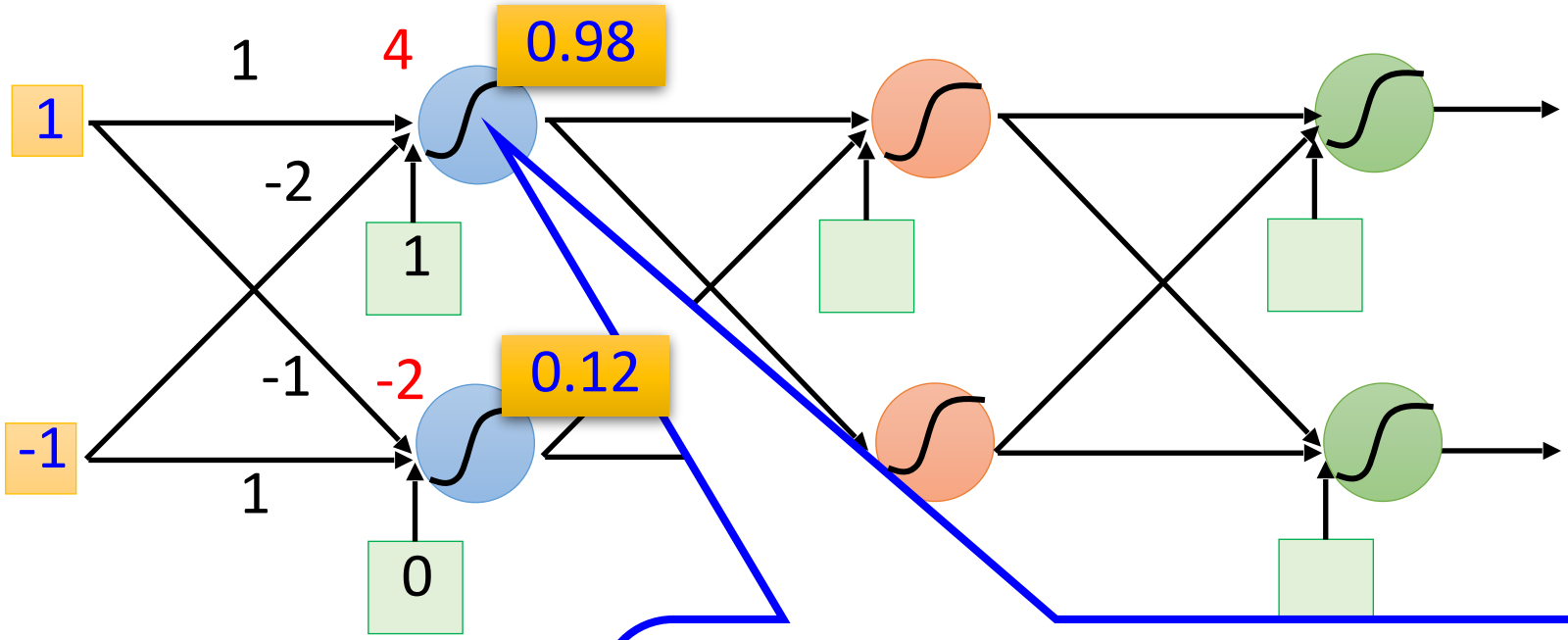


Neural Network



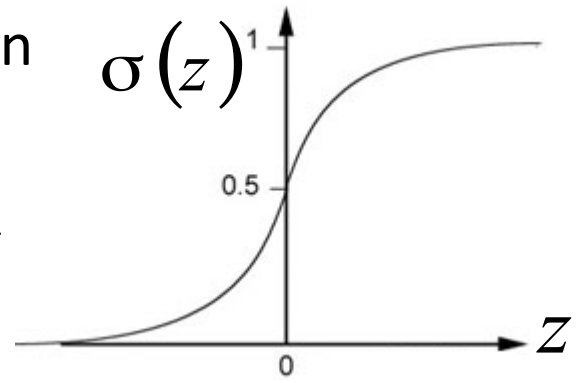
Deep means many hidden layers

Example of Neural Network



Sigmoid Function

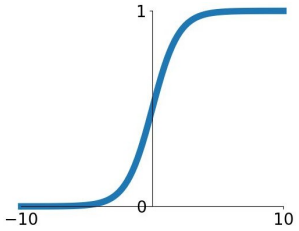
$$\sigma(z) = \frac{1}{1+e^{-z}}$$



Activation functions

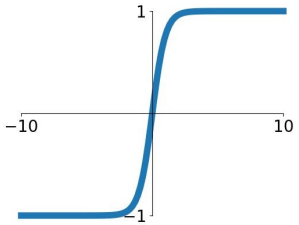
Sigmoid

$$\sigma(x) = \frac{1}{1+e^{-x}}$$



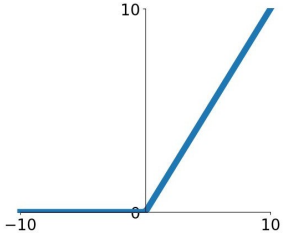
tanh

$$\tanh(x)$$



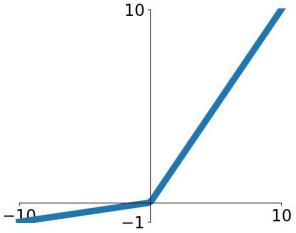
ReLU

$$\max(0, x)$$



Leaky ReLU

$$\max(0.1x, x)$$

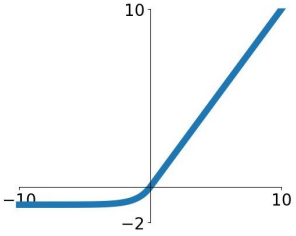


Maxout

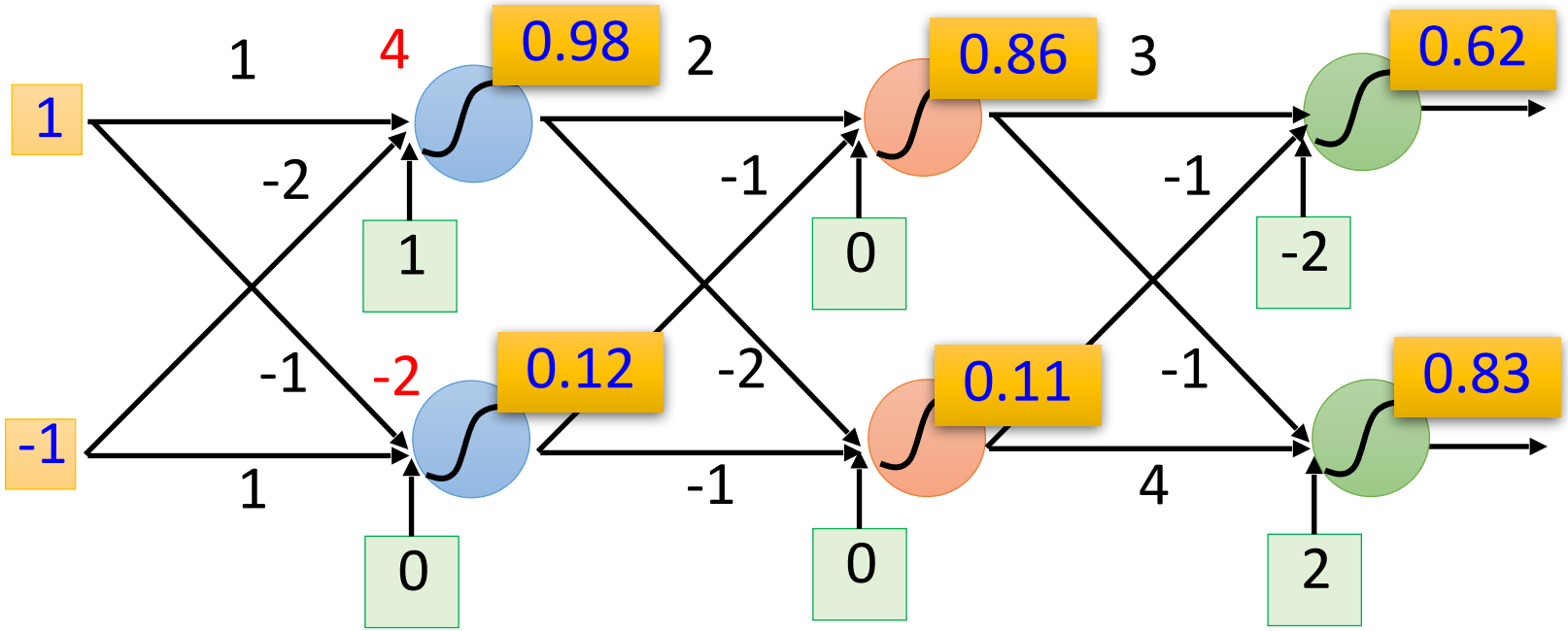
$$\max(w_1^T x + b_1, w_2^T x + b_2)$$

ELU

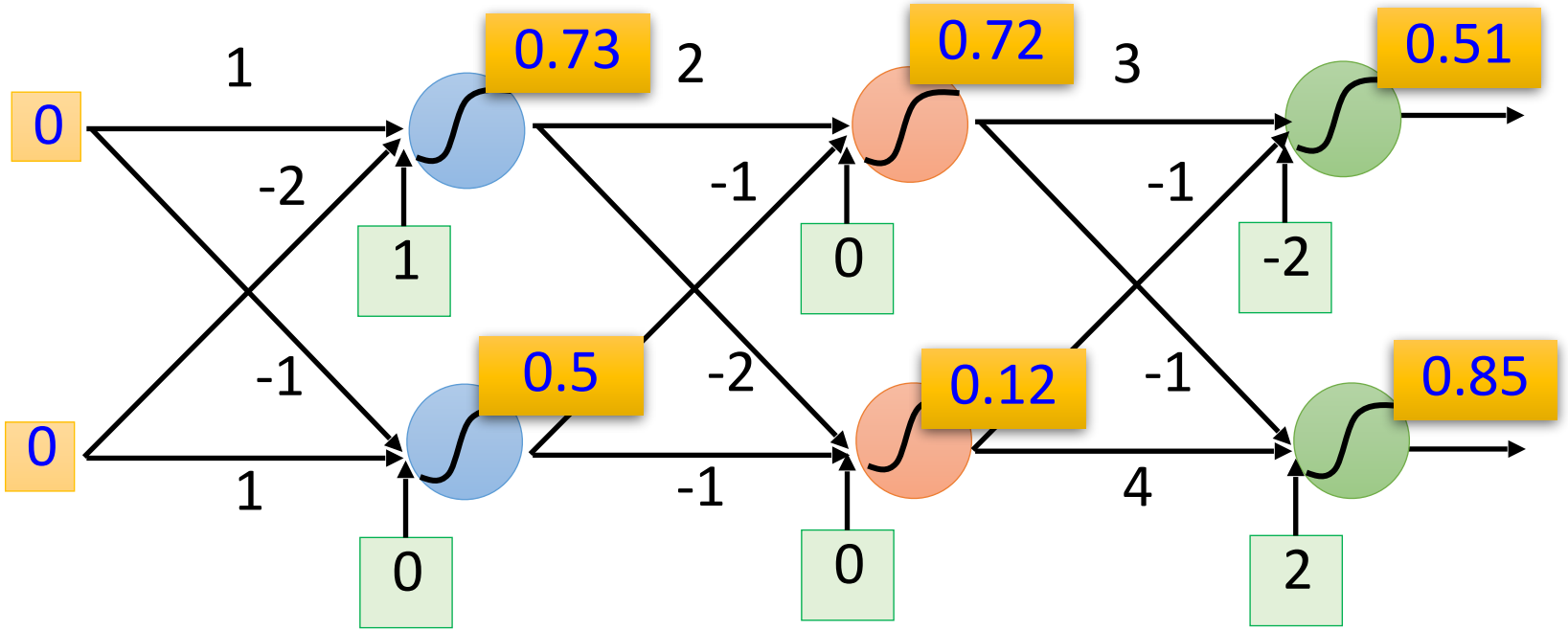
$$\begin{cases} x & x \geq 0 \\ \alpha(e^x - 1) & x < 0 \end{cases}$$



Example of Neural Network



Example of Neural Network

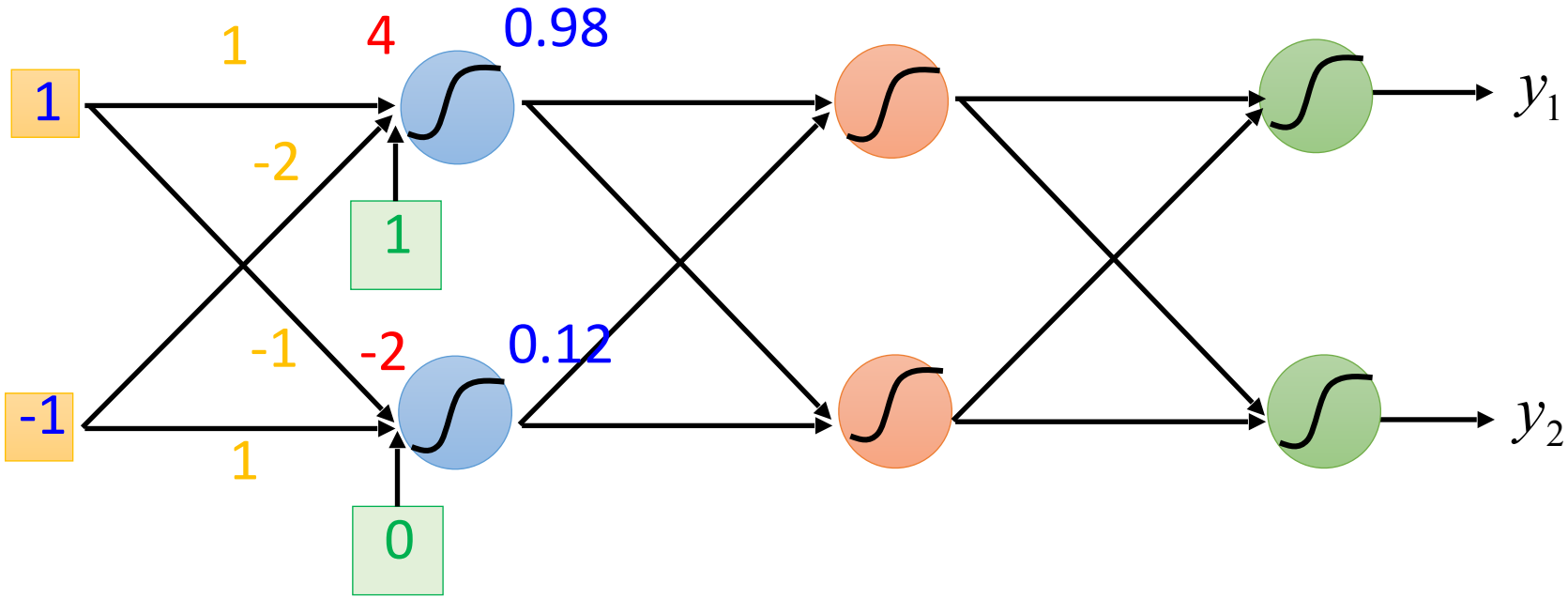


$f: R^2 \rightarrow R^2$

$$f\left(\begin{bmatrix}1\\-1\end{bmatrix}\right)=\begin{bmatrix}0.62\\0.83\end{bmatrix} \quad f\left(\begin{bmatrix}0\\0\end{bmatrix}\right)=\begin{bmatrix}0.51\\0.85\end{bmatrix}$$

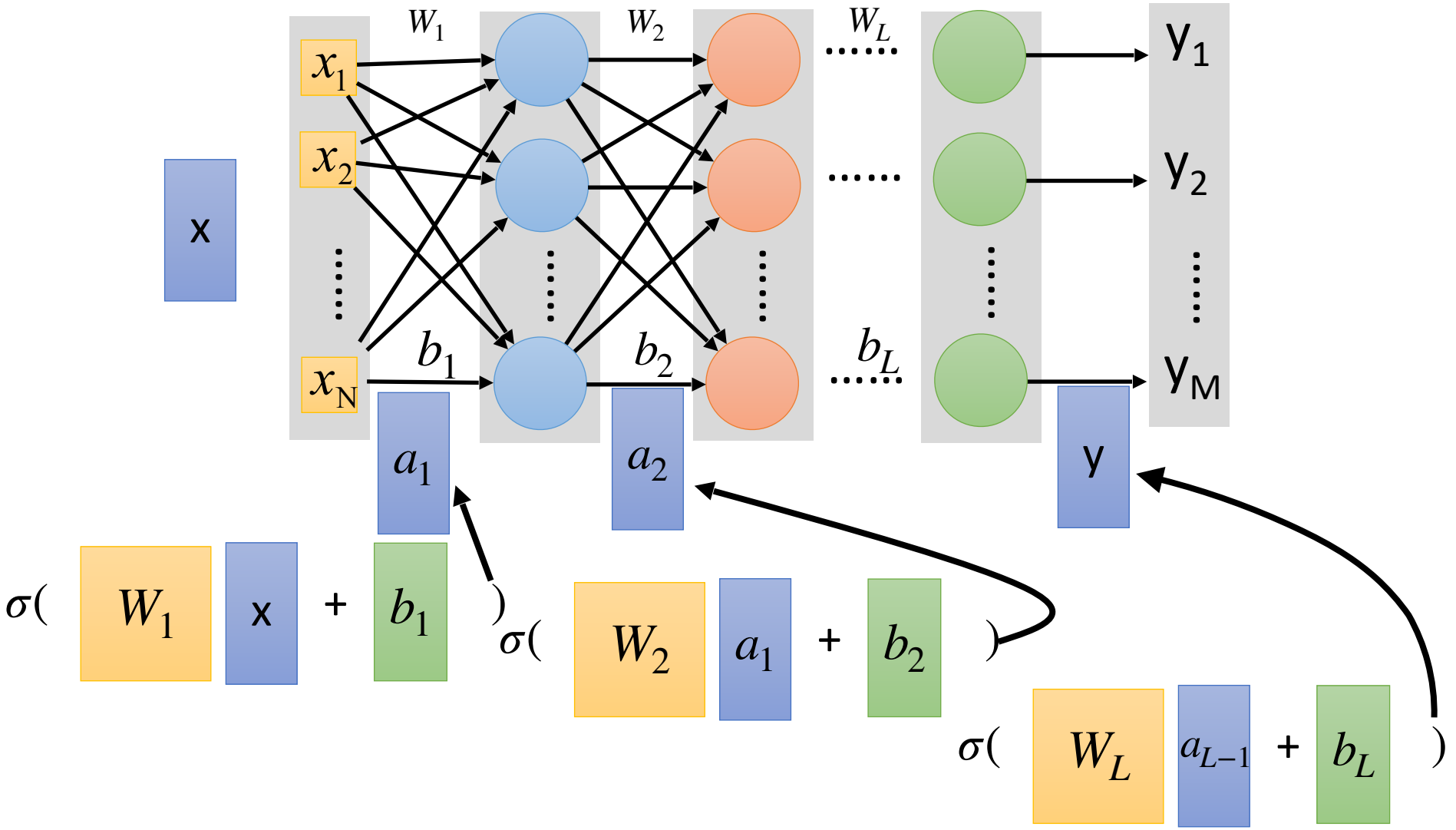
Different parameters define different function

Matrix Operation

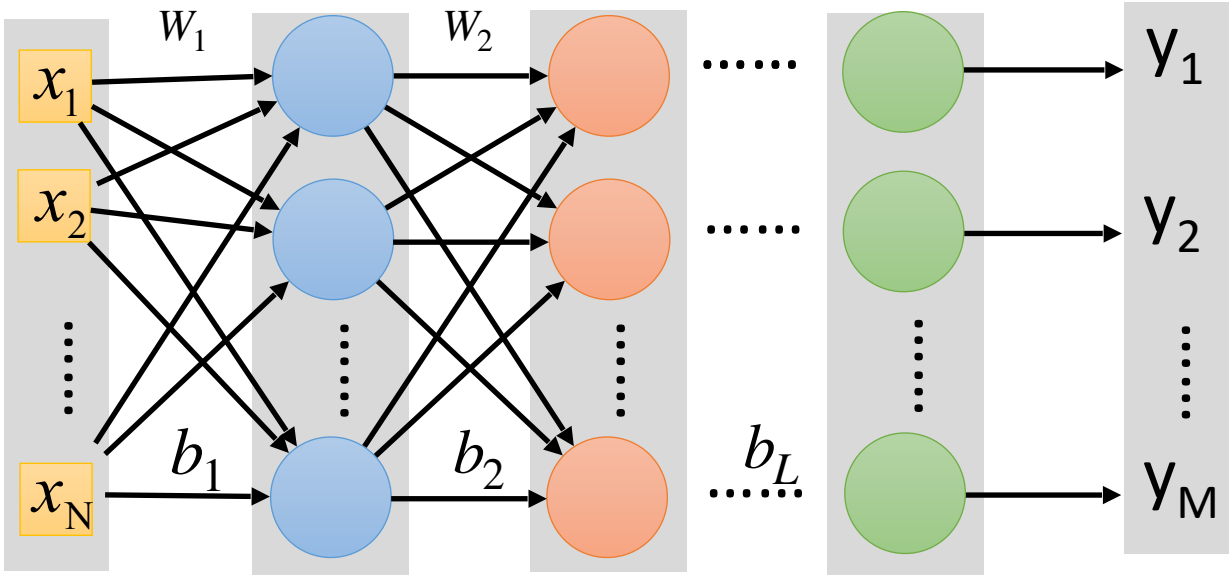


$$\sigma\left(\underbrace{\begin{bmatrix}1 & -2\\ -1 & 1\end{bmatrix}\begin{bmatrix}1\\ -1\end{bmatrix}+\begin{bmatrix}1\\ 0\end{bmatrix}}_{\begin{bmatrix}4\\ -2\end{bmatrix}}\right)=\begin{bmatrix}0.98\\ 0.12\end{bmatrix}$$

Neural Network



Neural Network



y = f(x)

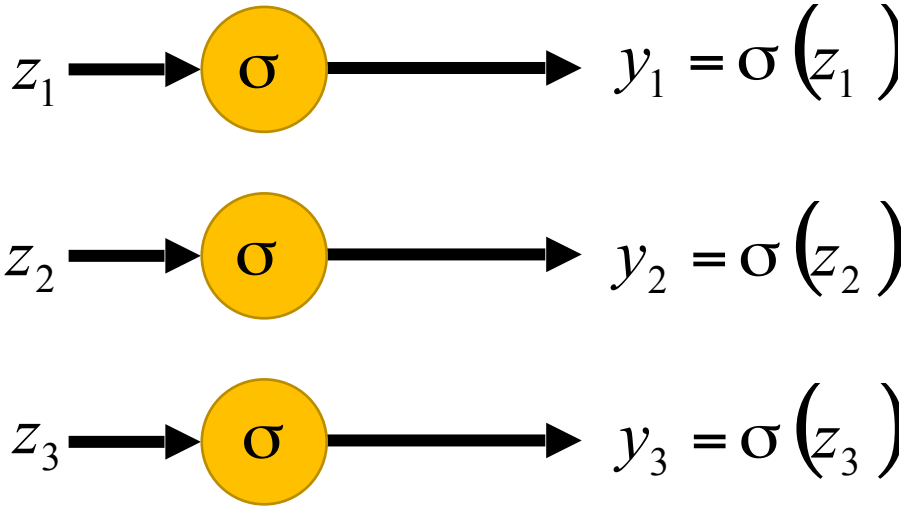
Using parallel computing techniques to speed up matrix operation

$$\sigma(W_L \cdots \ \sigma(W_2\ \sigma(W_1 x + b_1) + b_2) + \cdots b_L)$$

Softmax

- Softmax layer as the output layer

Ordinary Layer



In general, the output of network can be any value.

May not be easy to interpret

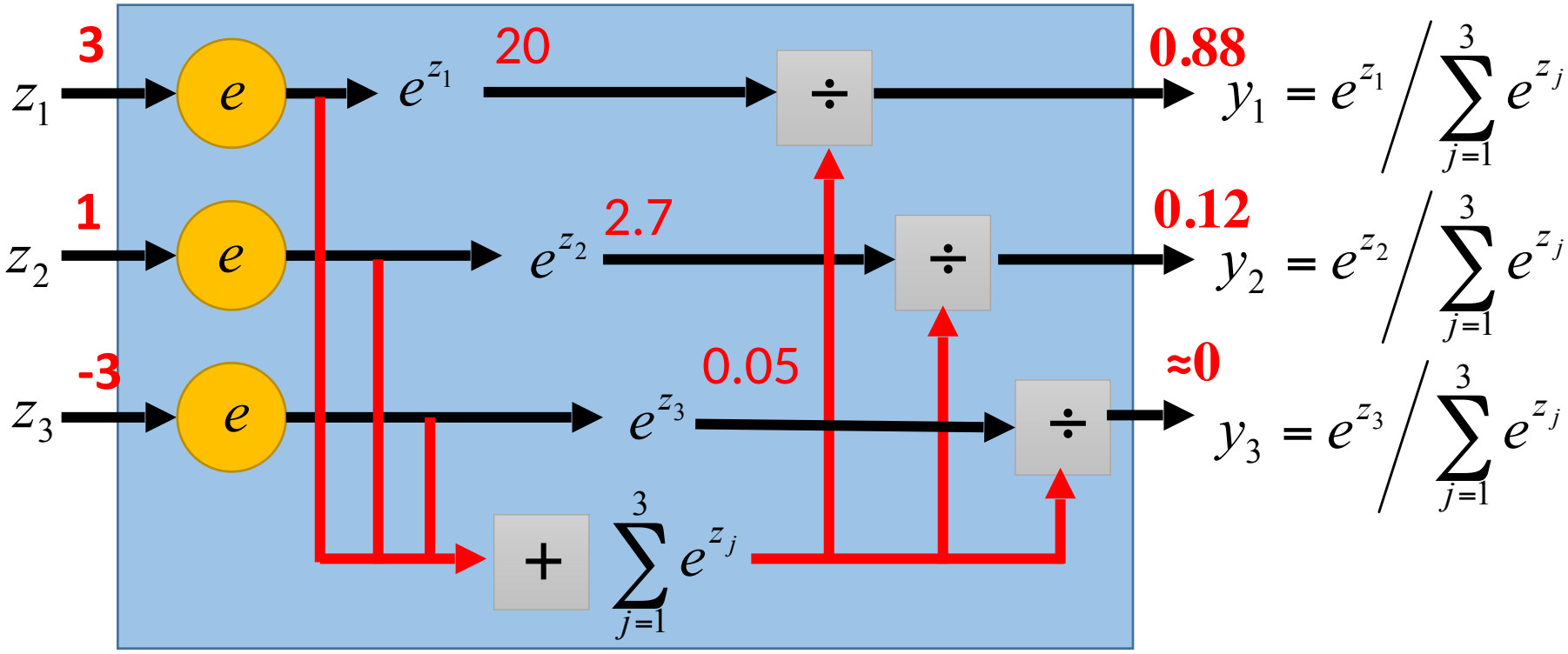
Softmax

- Softmax layer as the output layer

Probability:

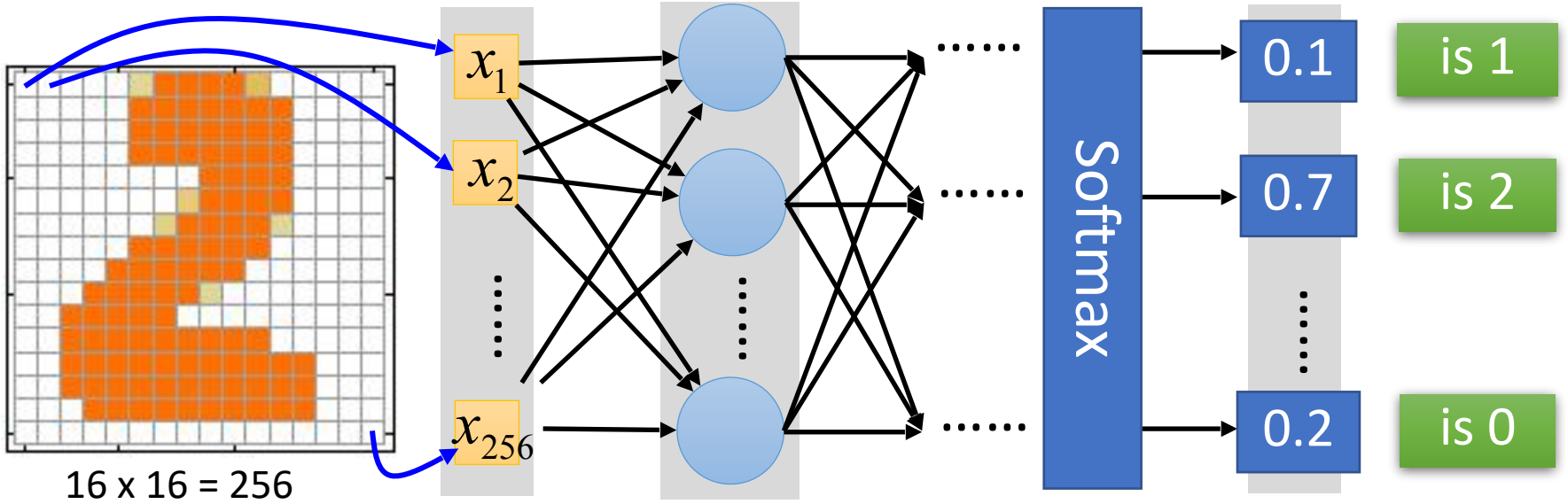
- $1 > y_i > 0$
- $\sum_i y_i = 1$

Softmax Layer



How to set network parameters

$$\theta = \{W_1, b_1, \ldots, W_n, b_n\}$$



16 x 16 = 256

Ink → 1

No ink → 0

Set the network parameters θ such that ……

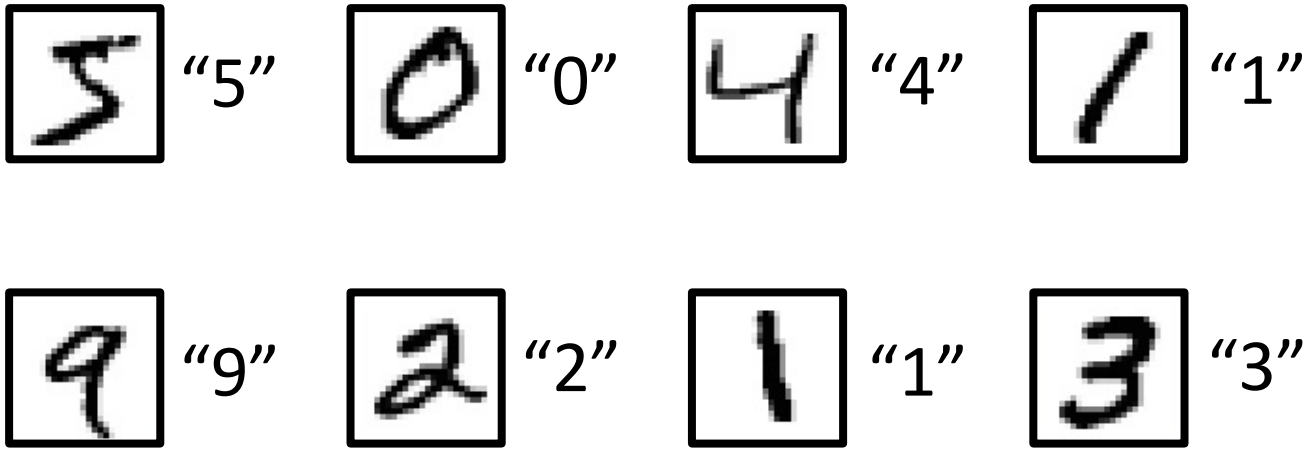
Input … m value

Input: … y_2 has the maximum value

How to let the neural network achieve this

Training Data

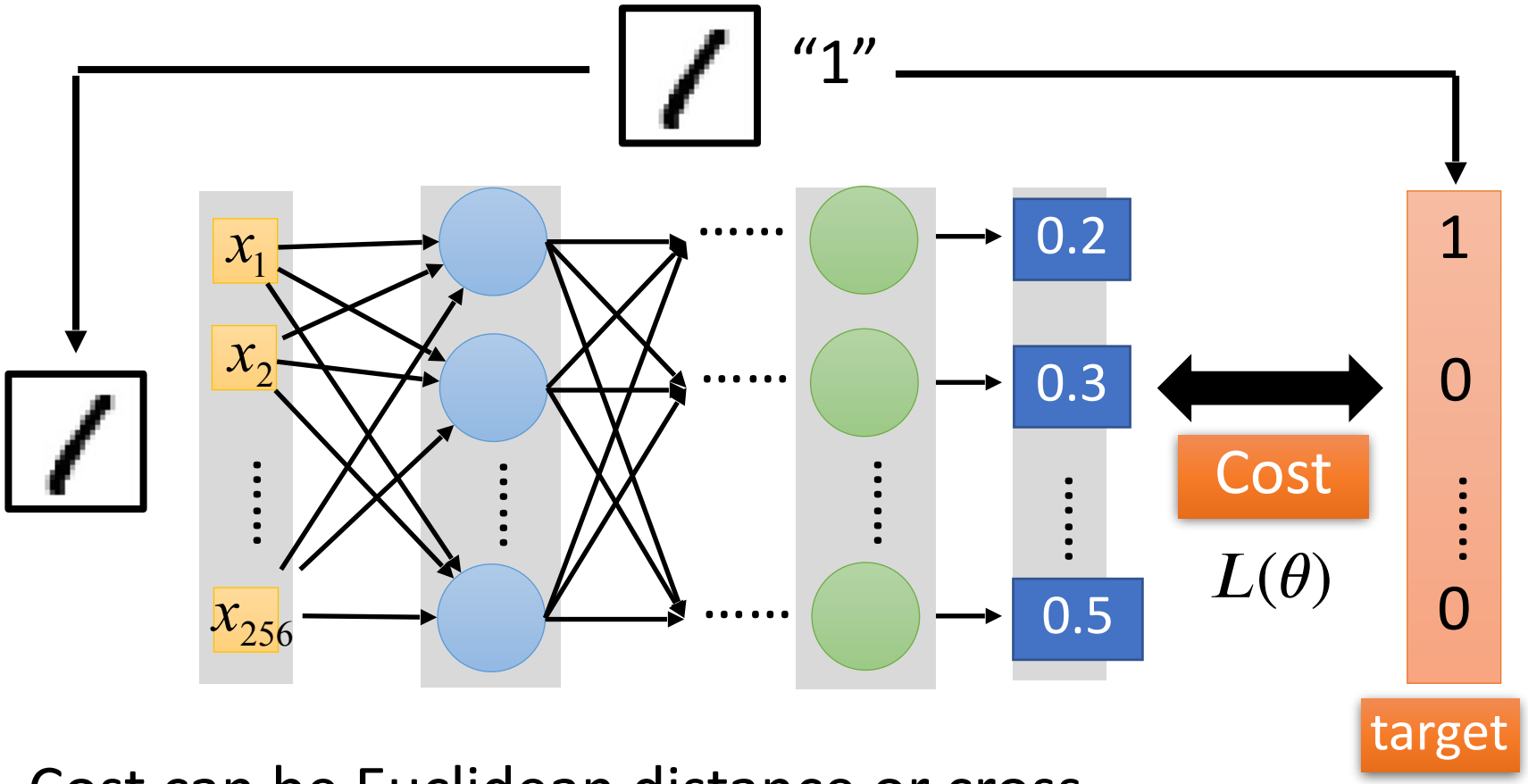
- Preparing training data: images and their labels



Using the training data to find the network parameters.

Cost

Given a set of network parameters θ, each example has a cost value.



Cost can be Euclidean distance or cross entropy of the network output and target

Soft-entropy Loss

The score of label category is larger than other categories:

$$score_{\text{label}} > score_j \text{ for any } j \neq \text{label}$$

How to set up a loss for this goal?

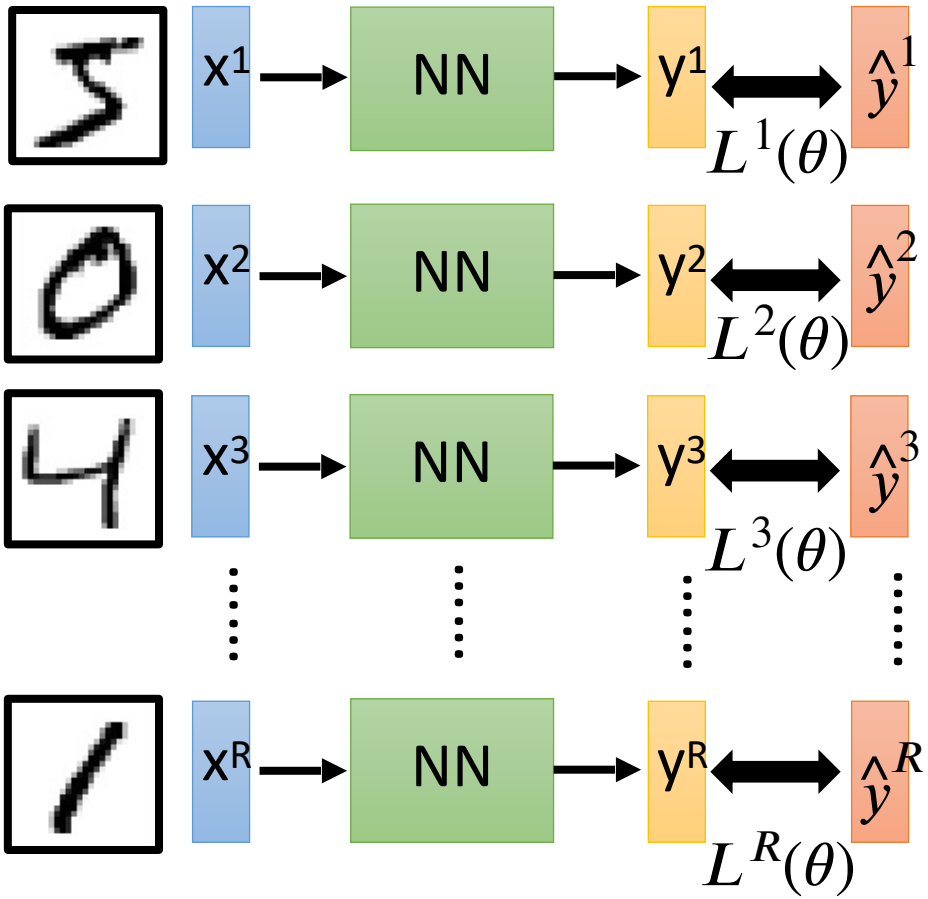
Soft-entropy Loss

Let $score_{label} = \frac{e^{f(x,W)_{label}}}{\sum_j e^{f(x,W)_j}}$

If we set $\ell(f(x; W, b), \text{label}) = -\log score_{\text{label}}$

Total Cost

For all training data …



Total Cost:

$$C(\theta) = \sum_{r=1}^{R} L^r(\theta)$$

How bad the network parameters θ is on this task

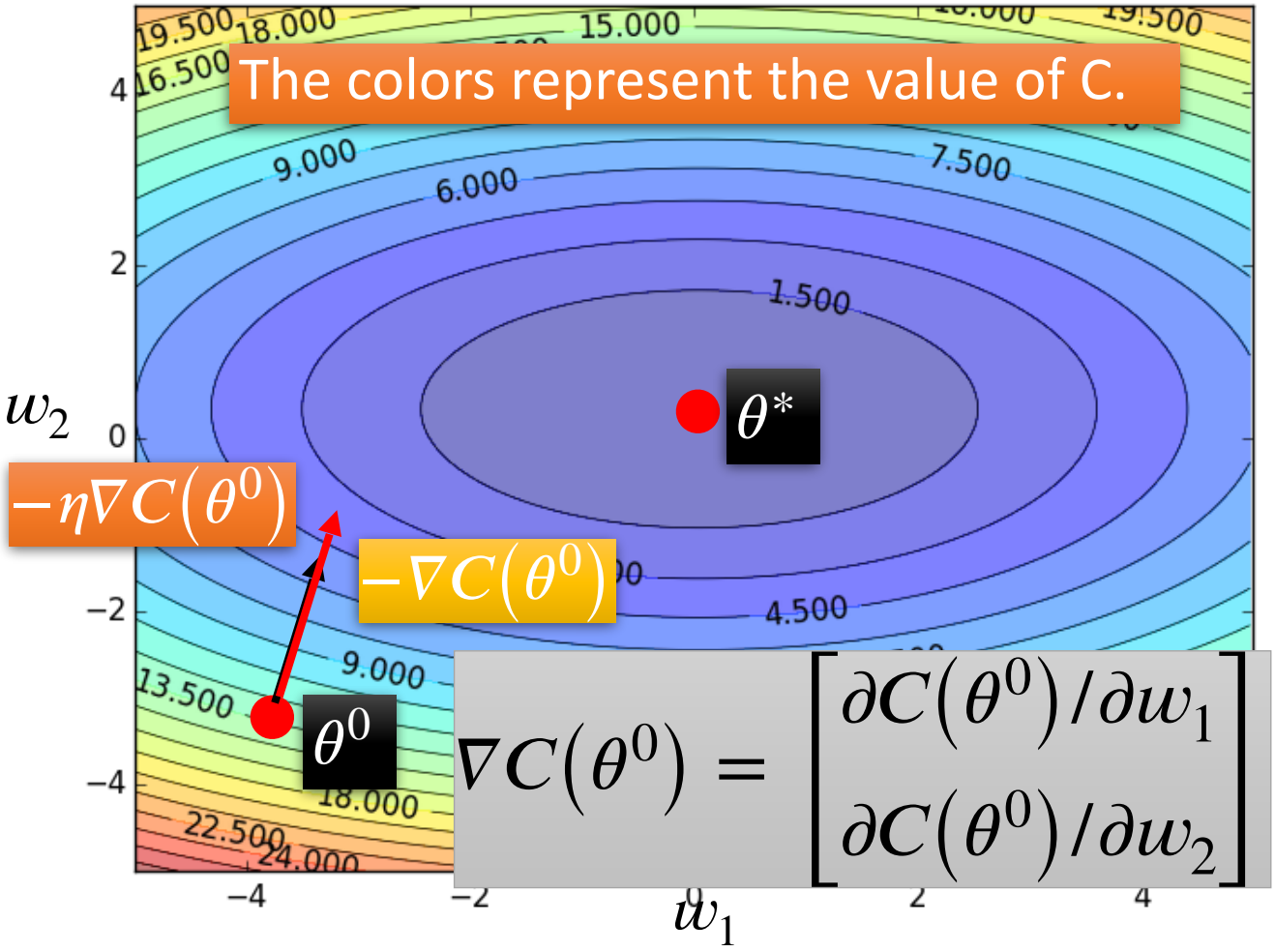
Find the network parameters θ^* that minimize this value

Gradient Descent

Assume there are only two parameters w_1 and w_2 in a network.

$$\theta = \{w_1, w_2\}$$

Error Surface



Randomly pick a starting point θ^0

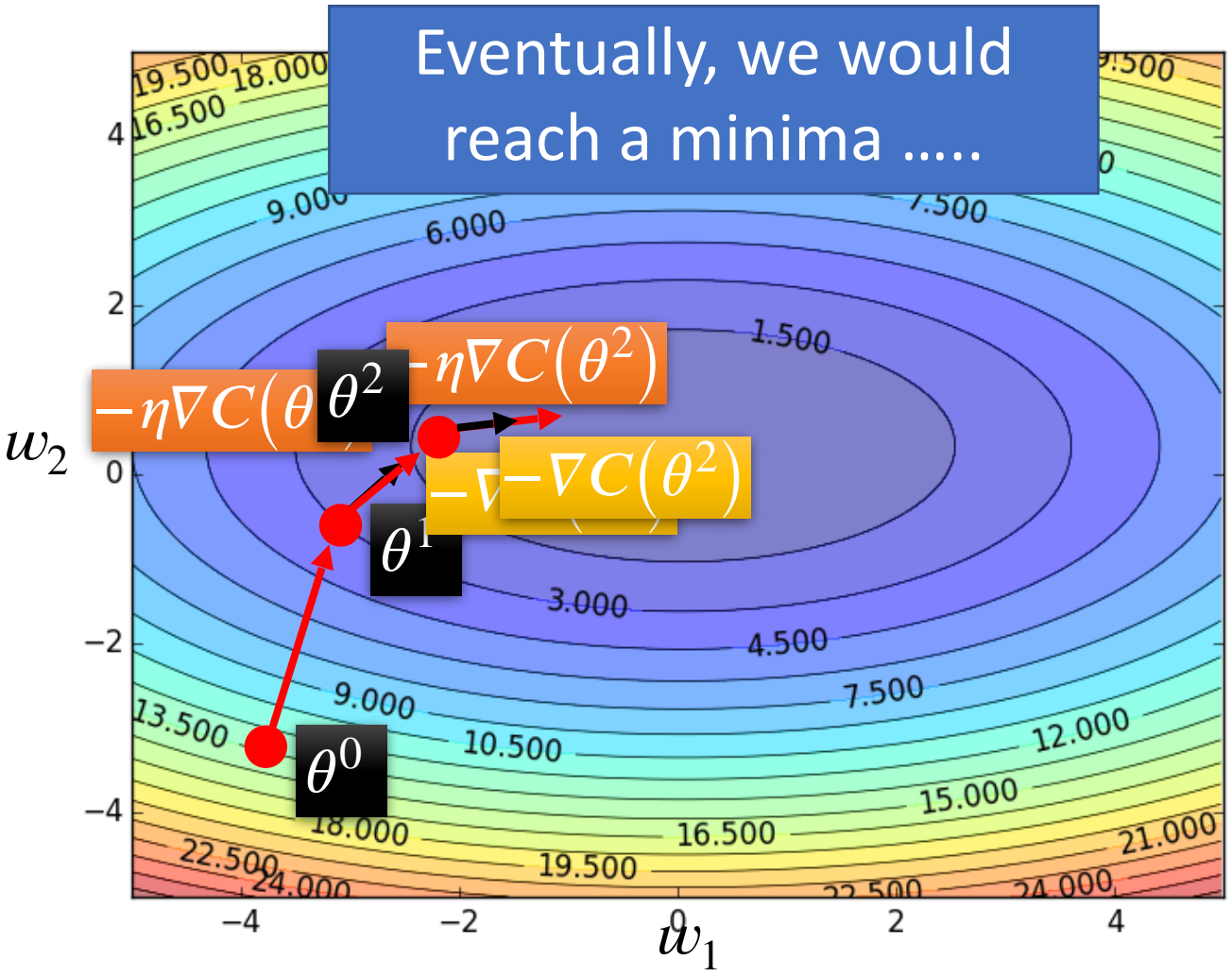
Compute the negative gradient at θ^0

$\Rightarrow -\nabla C(\theta^0)$

Times the learning rate η

$\Rightarrow -\eta\nabla C(\theta^0)$

Gradient Descent



Randomly pick a starting point θ^0

Compute the negative gradient at θ^0

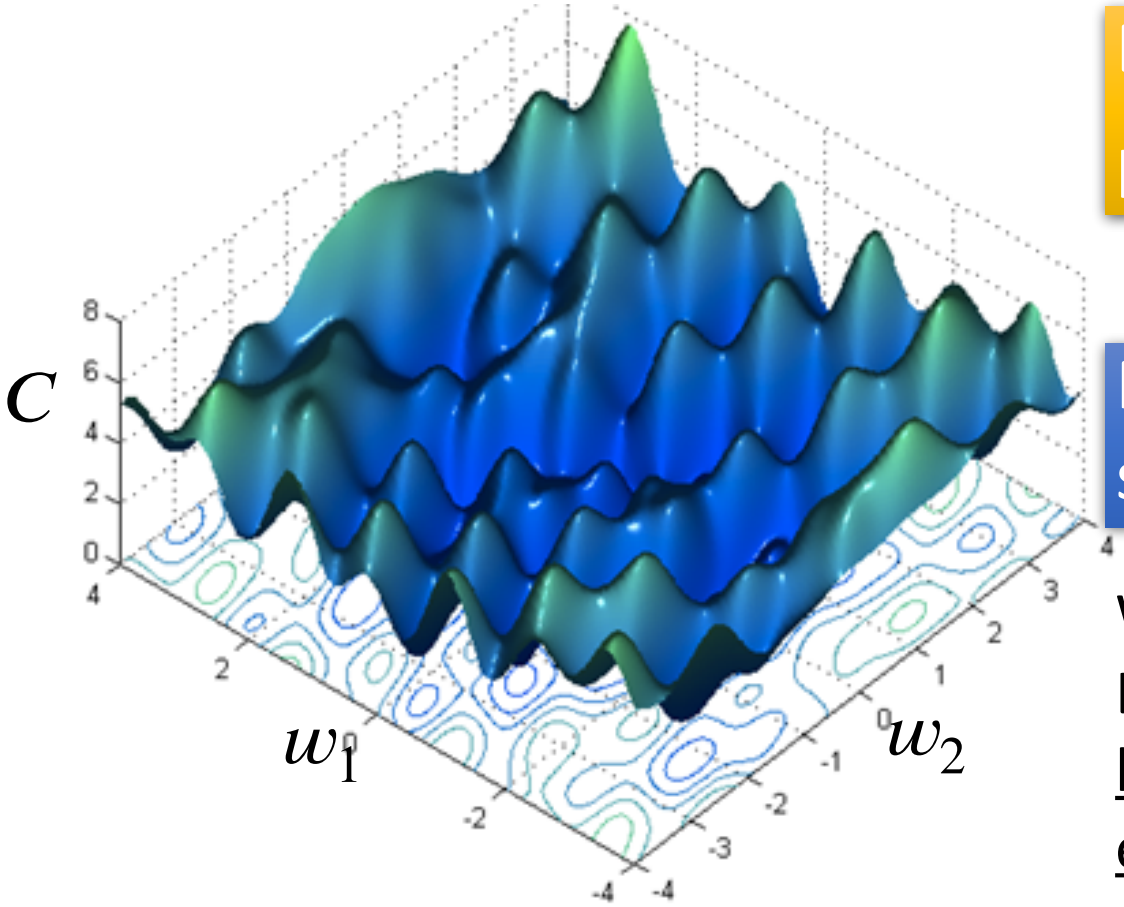
$\Rightarrow -\nabla C(\theta^0)$

Times the learning rate η

$\Rightarrow -\eta\nabla C(\theta^0)$

Local Minima

- Gradient descent never guarantee global minima

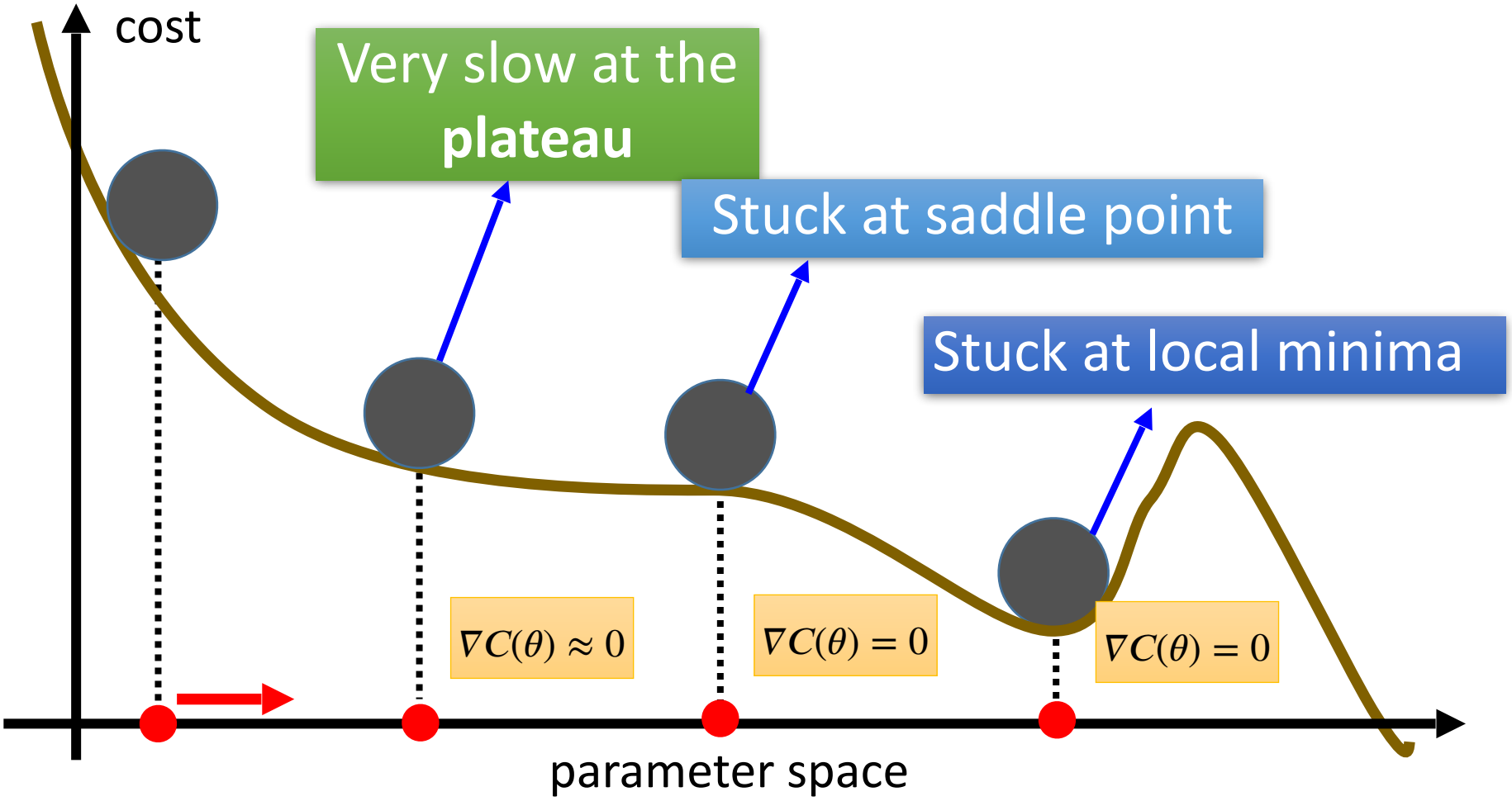


Different initial point θ^0

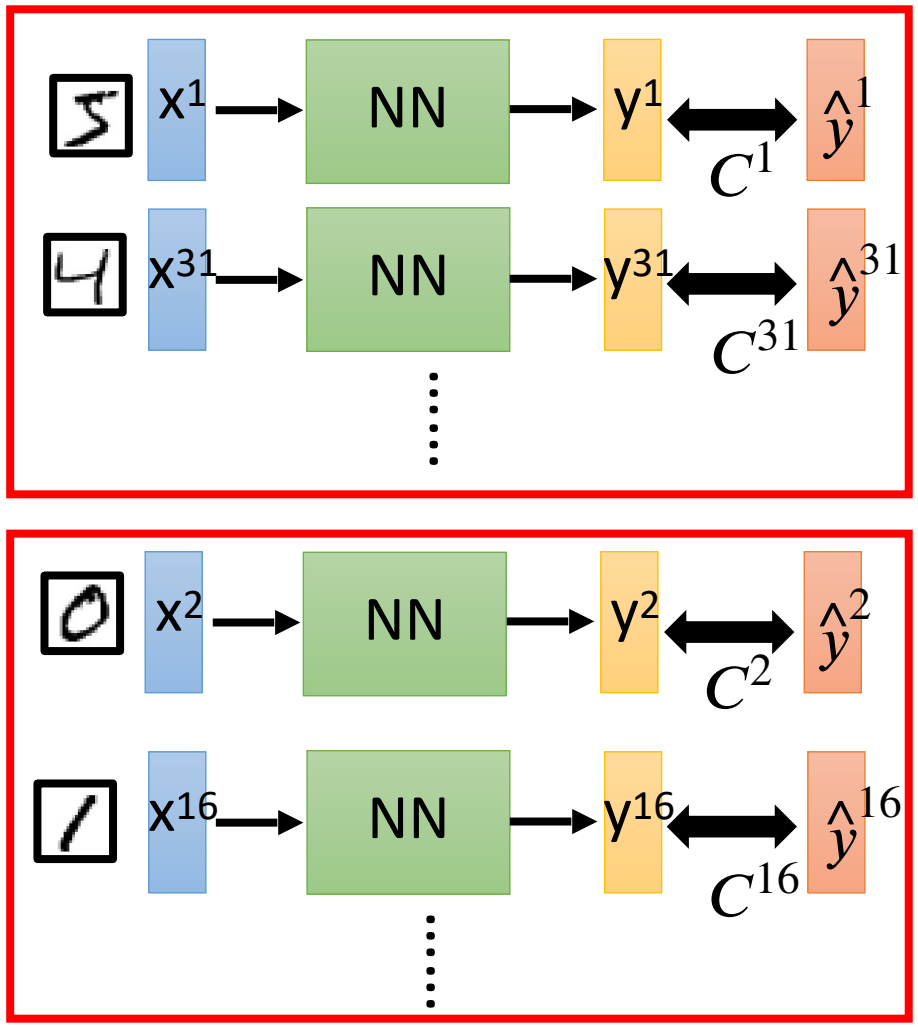
Reach different minima, so different results

Who is Afraid of Non-Convex Loss Functions?
http://videolectures.net/eml07_lecun_wia/

Besides local minima ……



Mini-batch



- Randomly initialize θ^0
- Pick the 1st batch

$$C = C^1 + C^{31} + \cdots$$

$$\theta^1 \leftarrow \theta^0 - \eta \nabla C(\theta^0)$$

- Pick the 2nd batch

$$C = C^2 + C^{16} + \cdots$$

$$\theta^2 \leftarrow \theta^1 - \eta \nabla C(\theta^1)$$

⋮

- Until all mini-batches have been picked

one epoch

Repeat the above process

Backpropagation

- A network can have millions of parameters.
  - Backpropagation is the way to compute the gradients efficiently (not today)
  - Ref: http://speech.ee.ntu.edu.tw/~tlkagk/courses/MLDS_2015_2/Lecture/DNN%20backprop.ecm.mp4/index.html
- Many toolkits can compute the gradients automatically

theano

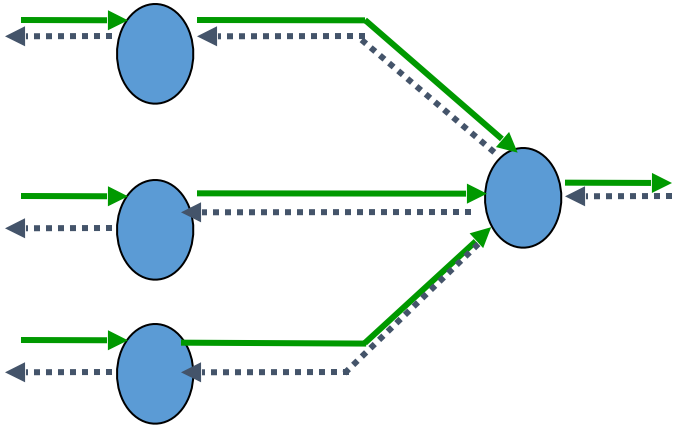
TensorFlow

Ref: http://speech.ee.ntu.edu.tw/~tlkagk/courses/MLDS_2015_2/Lecture/Theano%20DNN.ecm.mp4/index.html

Back Propagation

- Back-propagation training algorithm

Network activation
Forward Step



Error propagation
Backward Step

- Backprop adjusts the weights of the NN in order to minimize the network total error.

Next: Convolutional Neural Networks

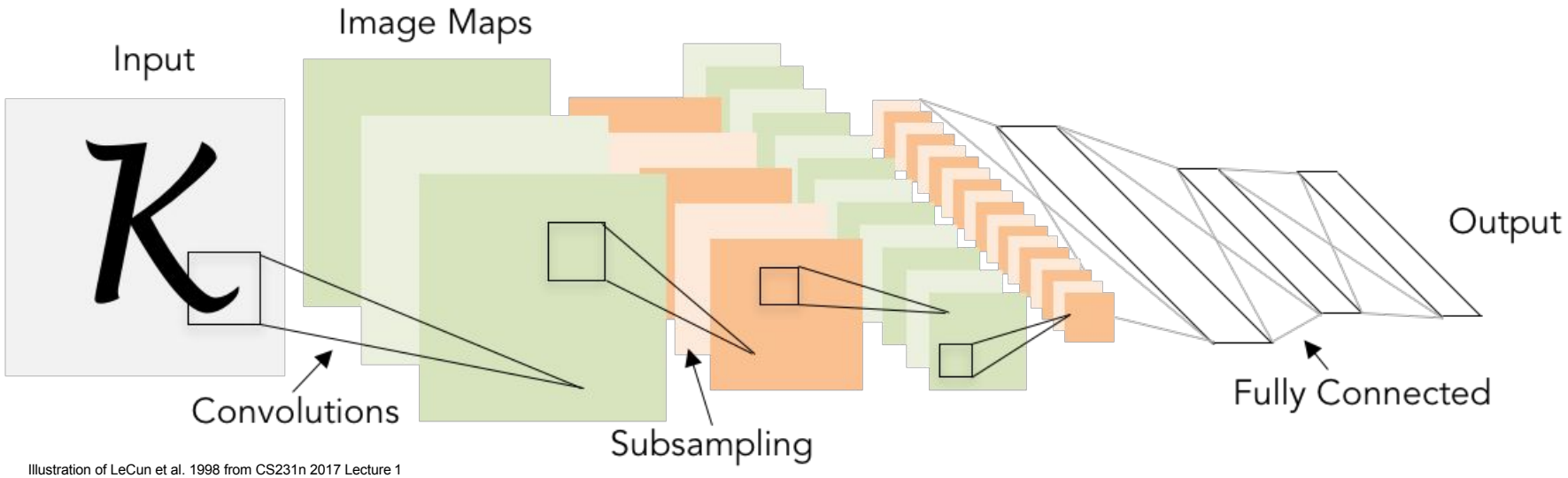
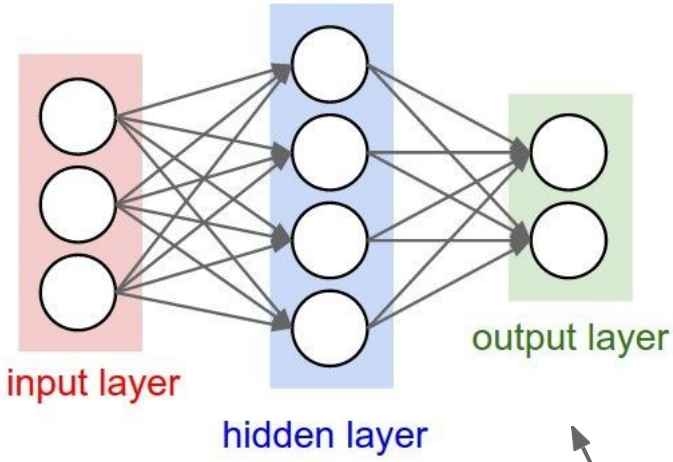
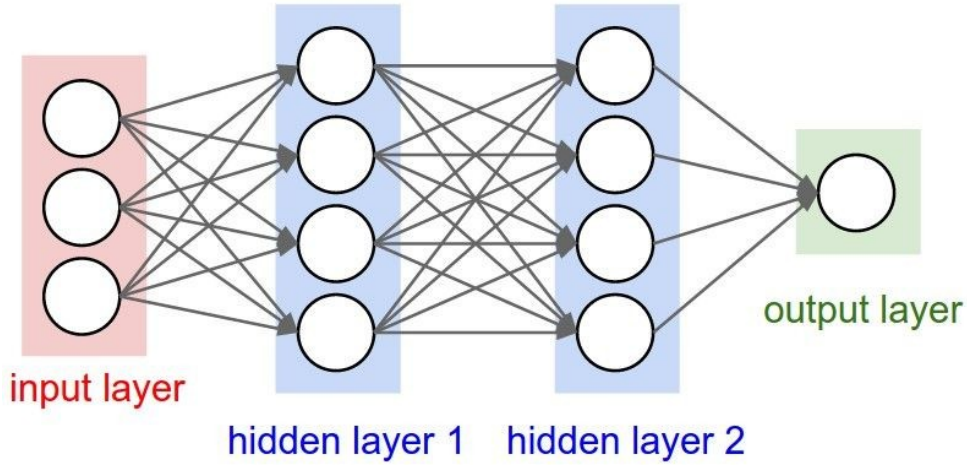


Illustration of LeCun et al. 1998 from CS231n 2017 Lecture 1

Neural networks: Architectures





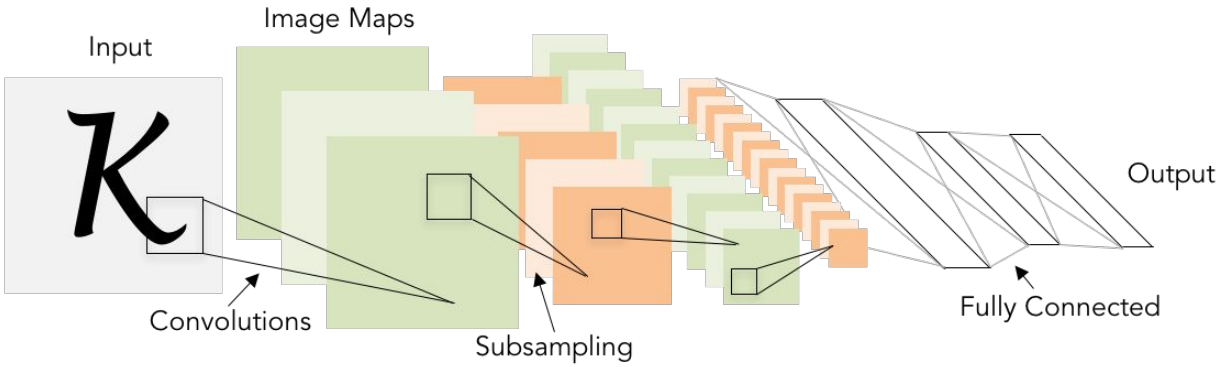
"2-layer Neural Net", or
"1-hidden-layer Neural Net"

"3-layer Neural Net", or
"2-hidden-layer Neural Net"

"Fully-connected" layers

Gradient-based learning applied to document recognition
[LeCun, Bottou, Bengio, Haffner 1998]

A bit of history:



LeNet-5

A bit of history:

ImageNet Classification with Deep Convolutional Neural Networks

[Krizhevsky, Sutskever, Hinton, 2012]

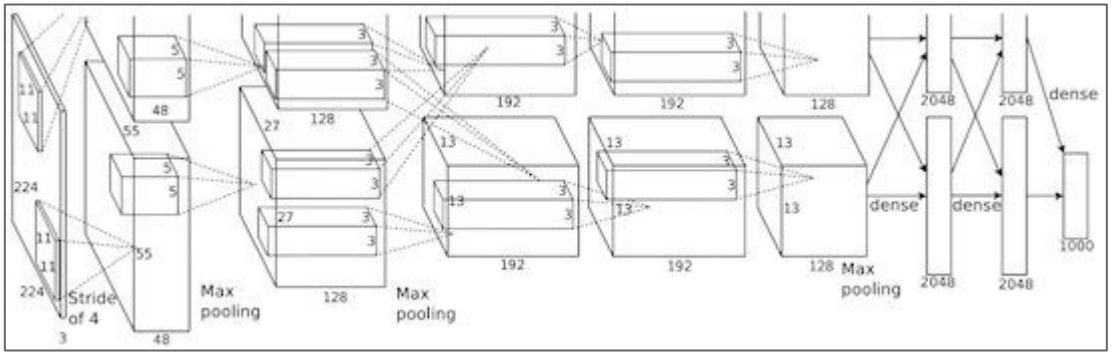


Figure copyright Alex Krizhevsky, Ilya Sutskever, and Geoffrey Hinton, 2012. Reproduced with permission.

"AlexNet"

Fast-forward to today: ConvNets are everywhere

Photo by Lane McIntosh. Copyright CS231n 2017.

self-driving cars

NVIDIA Tesla line
(these are the GPUs on rye01.stanford.edu)

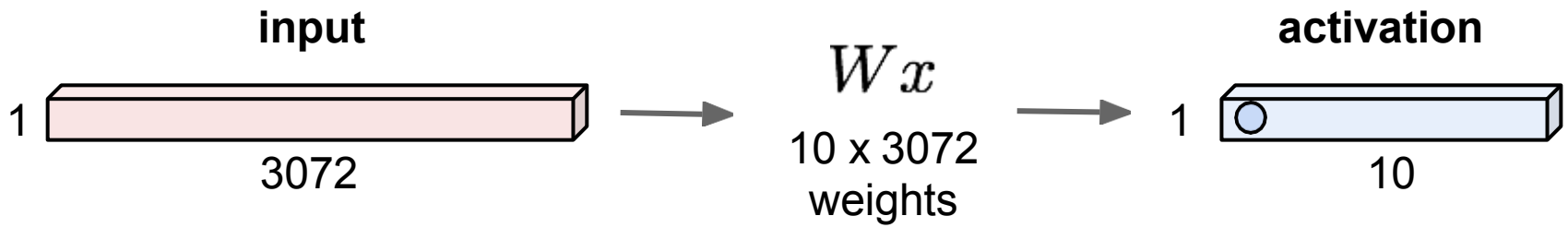
Note that for embedded systems a typical setup would involve NVIDIA Tegras, with integrated GPU and ARM-based CPU cores.

Convolutional Neural Networks

(First without the brain stuff)

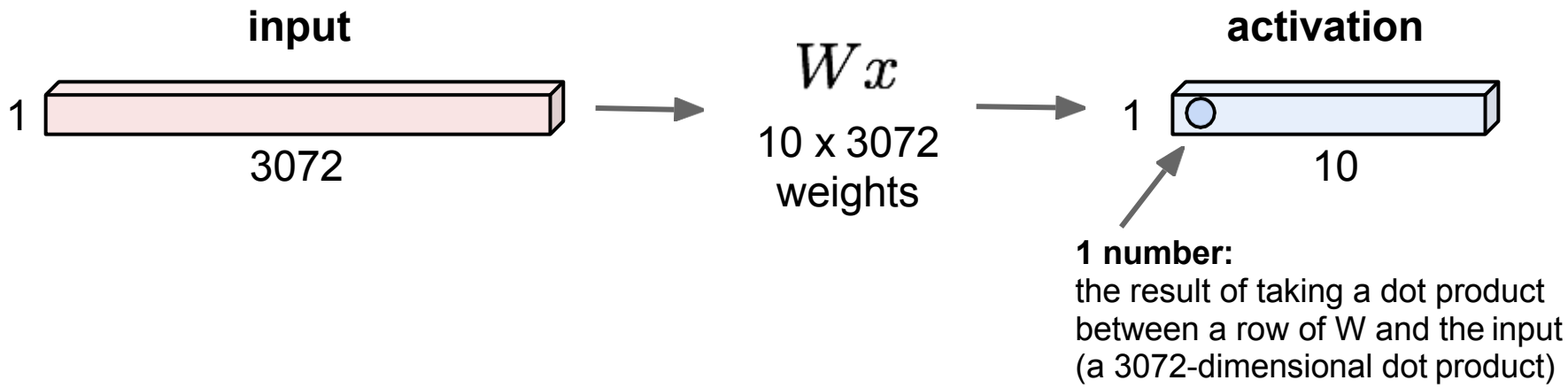
Fully Connected Layer

32x32x3 image -> stretch to 3072 x 1



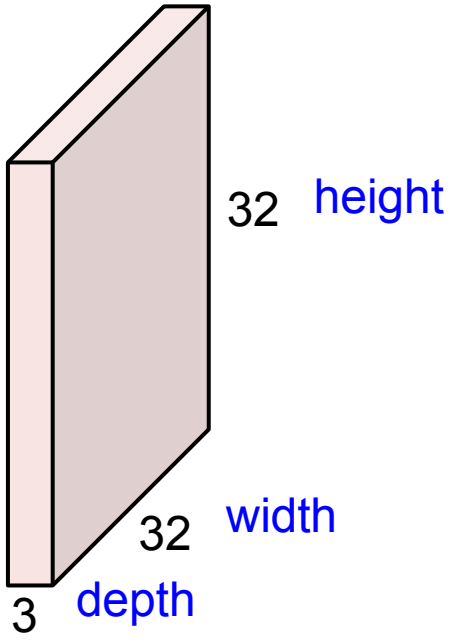
Fully Connected Layer

32x32x3 image -> stretch to 3072 x 1



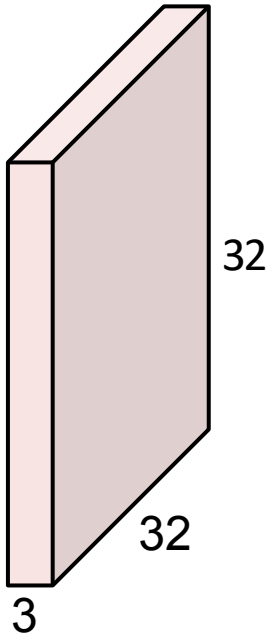
Convolution Layer

32x32x3 image -> preserve spatial structure

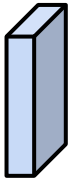


Convolution Layer

32x32x3 image



5x5x3 filter

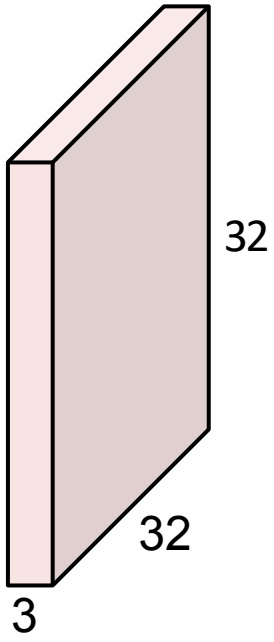


- **Convolve** the filter with the image
- i.e. "slide over the image spatially, computing dot products"

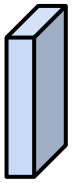
Convolution Layer

Filters always extend the full depth of the input volume

32x32x3 image

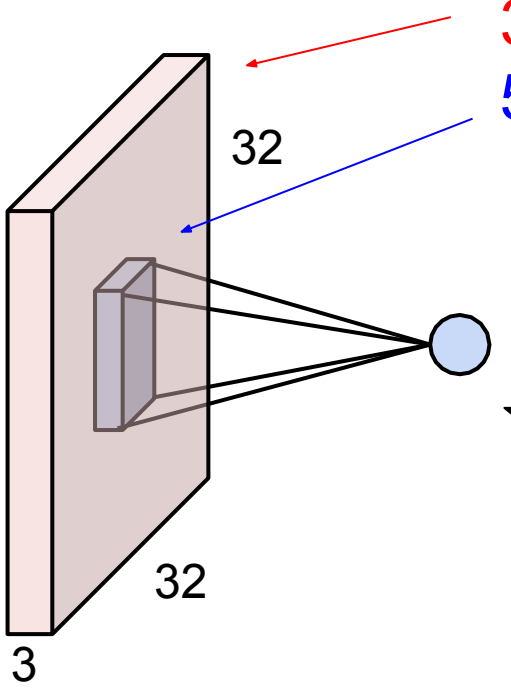


5x5x3 filter



- **Convolve** the filter with the image
- i.e. "slide over the image spatially, computing dot products"

Convolution Layer



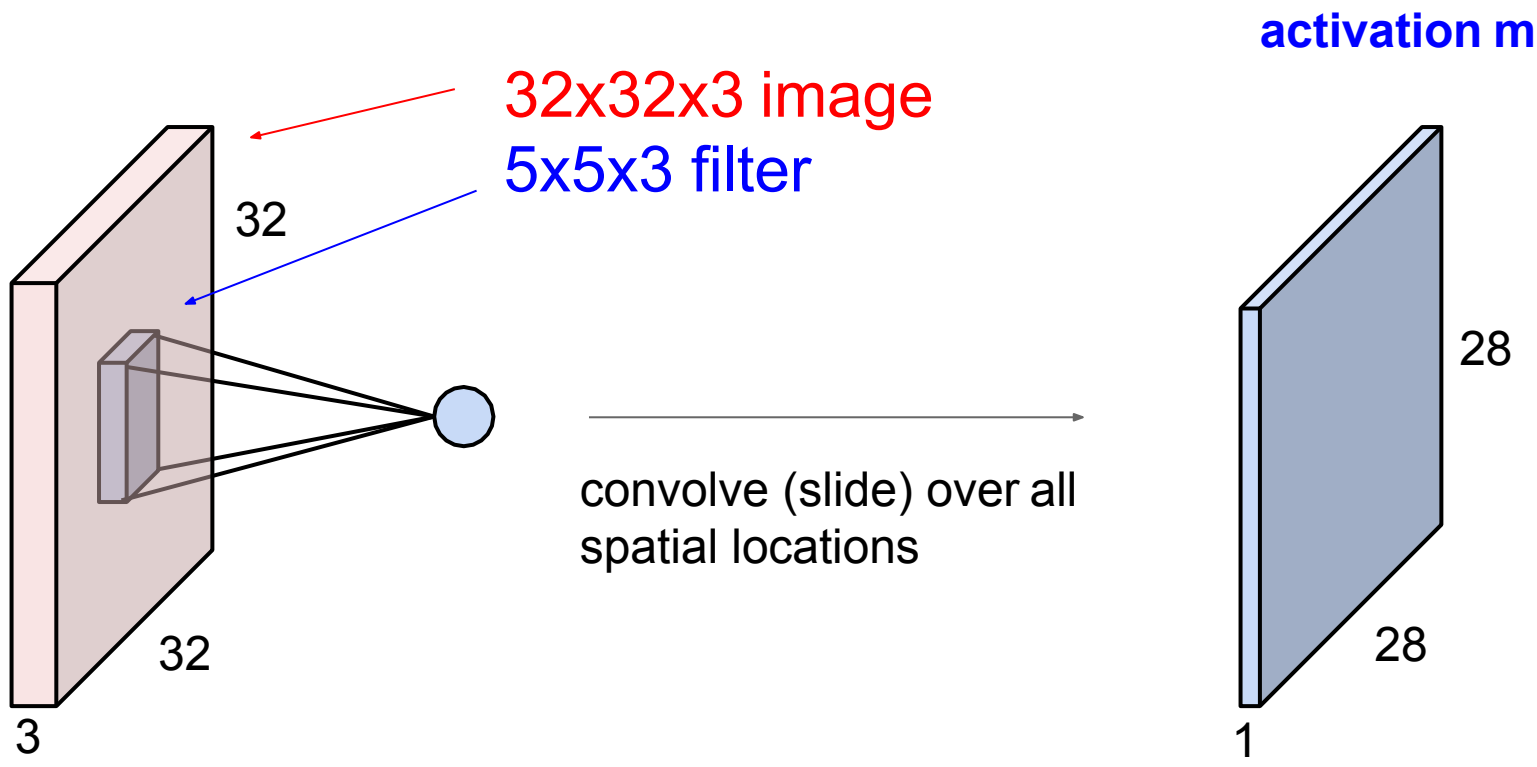
32x32x3 image

5x5x3 filter w

1 number:
the result of taking a dot product between the filter and a small 5x5x3 chunk of the image (i.e. 5*5*3 = 75-dimensional dot product + bias)

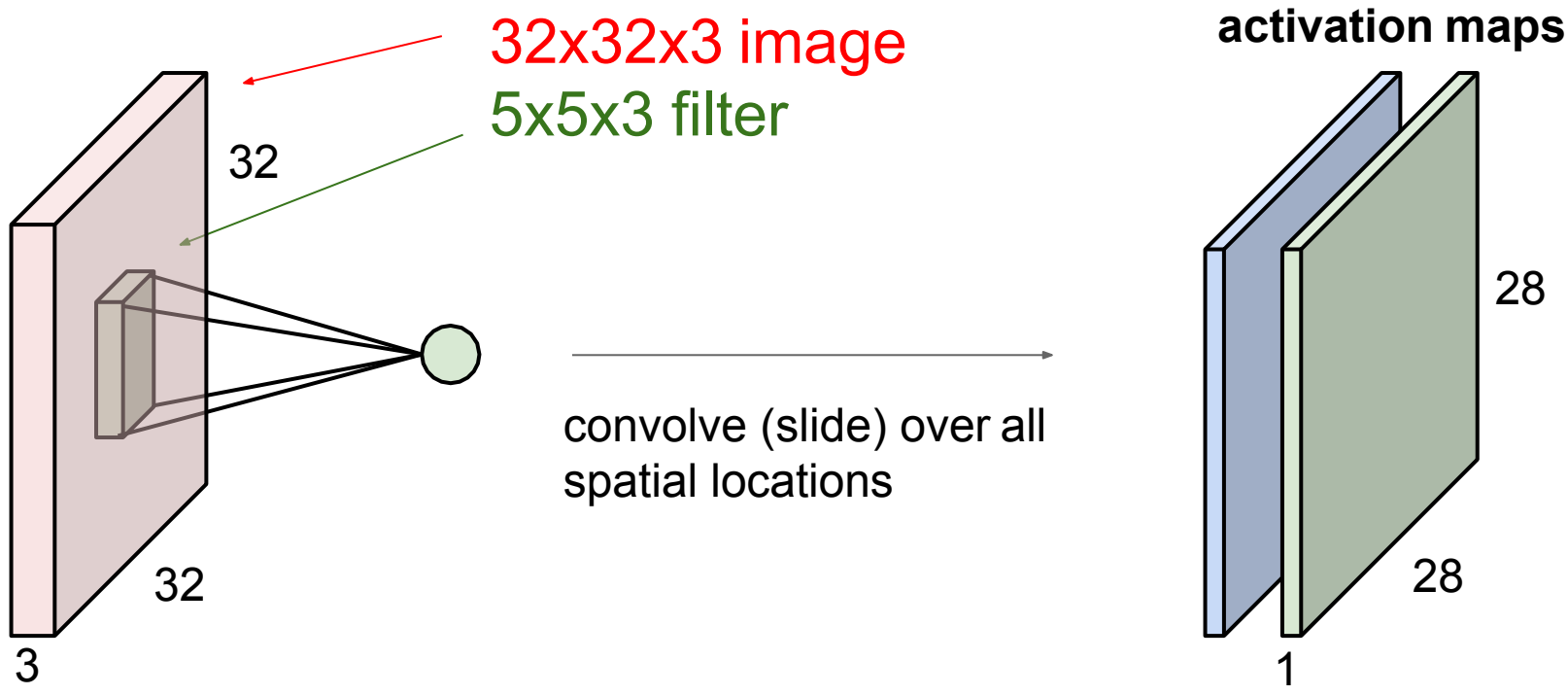
$$w^T x + b$$

Convolution Layer

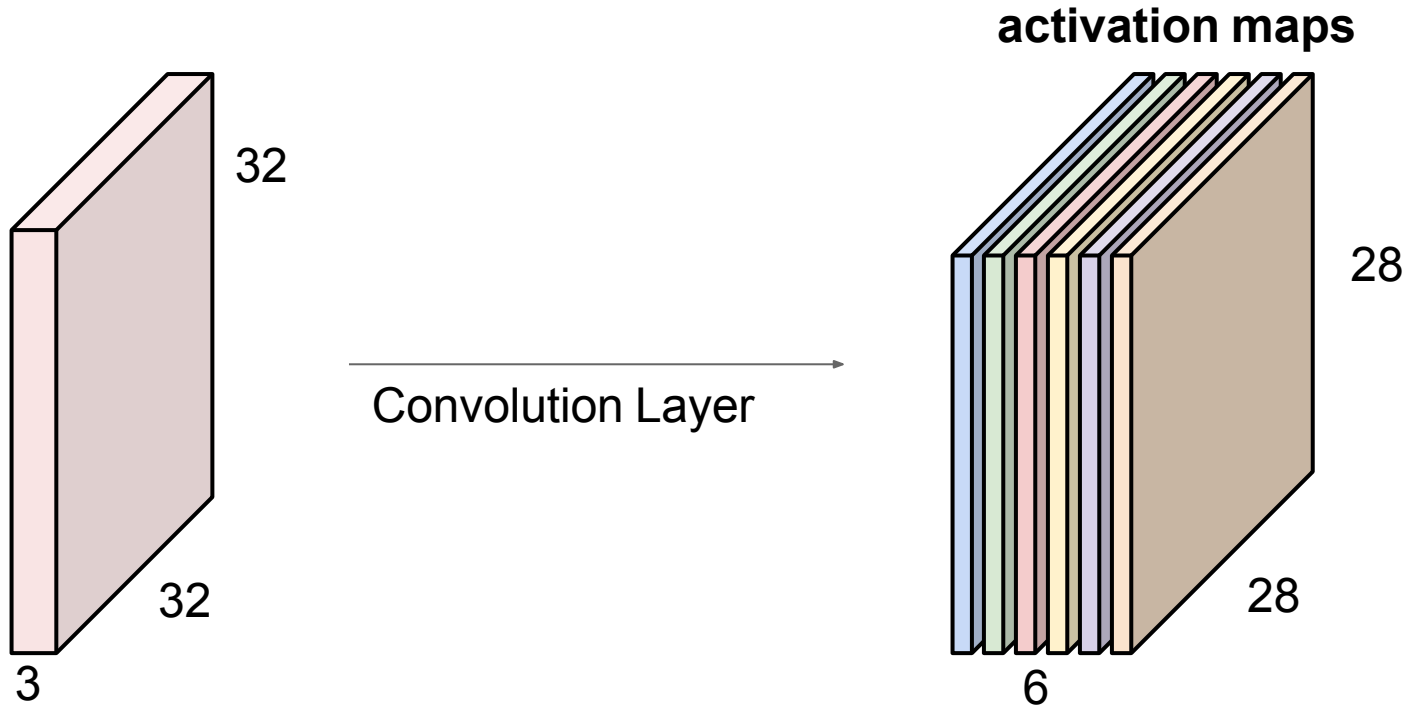


Convolution Layer

consider a second, green filter

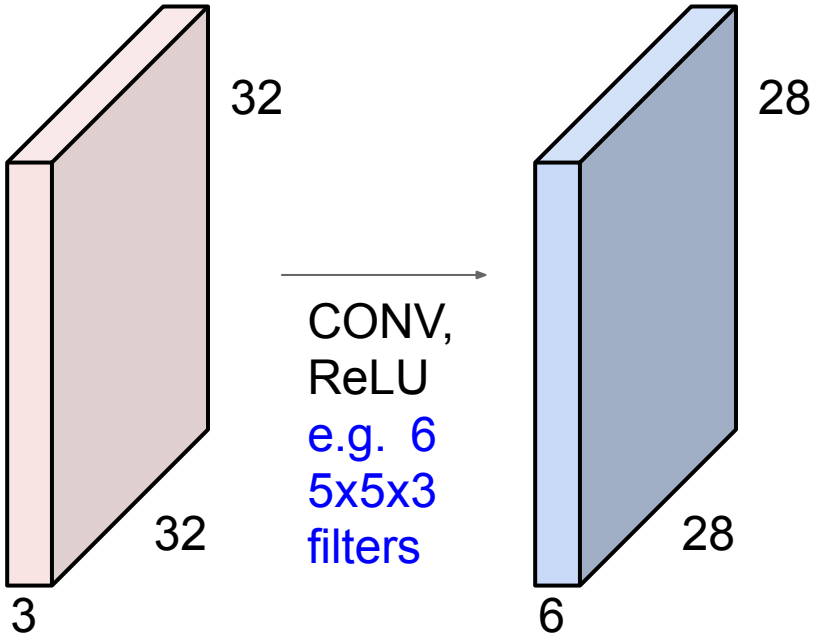


For example, if we had 6 5x5 filters, we'll get 6 separate activation maps:

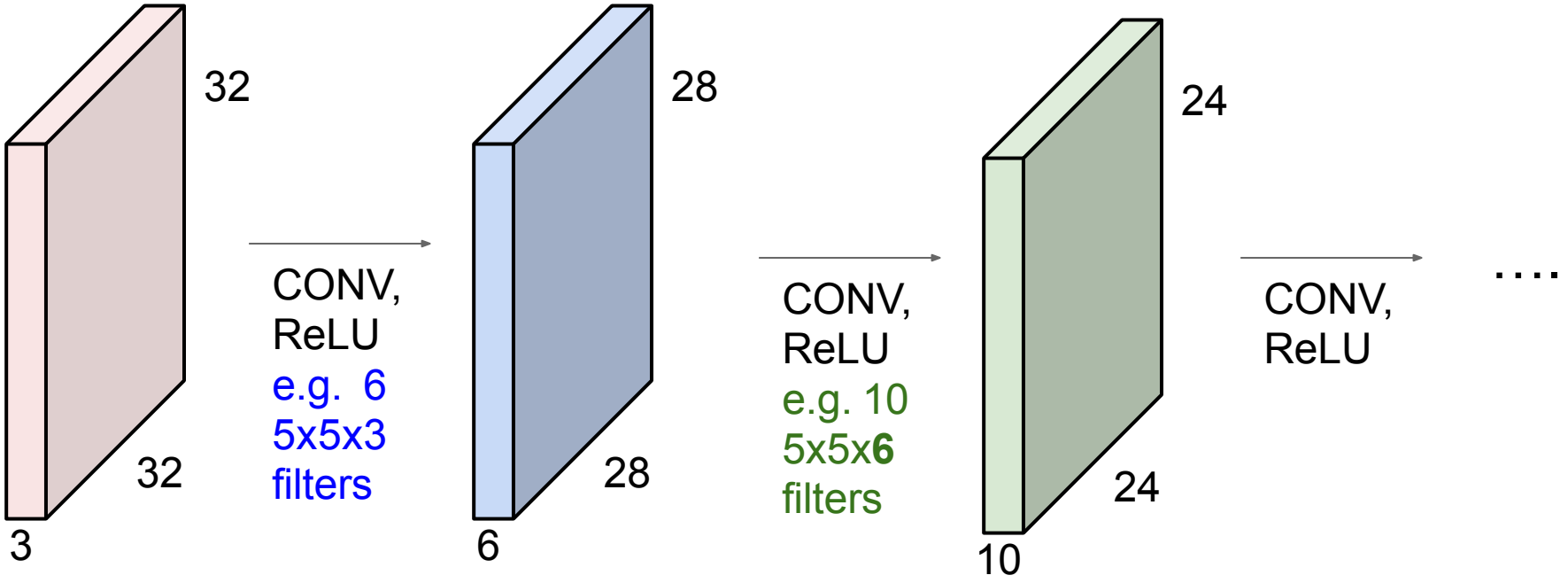


We stack these up to get a "new image" of size 28x28x6!

Preview: ConvNet is a sequence of Convolution Layers, interspersed with activation functions



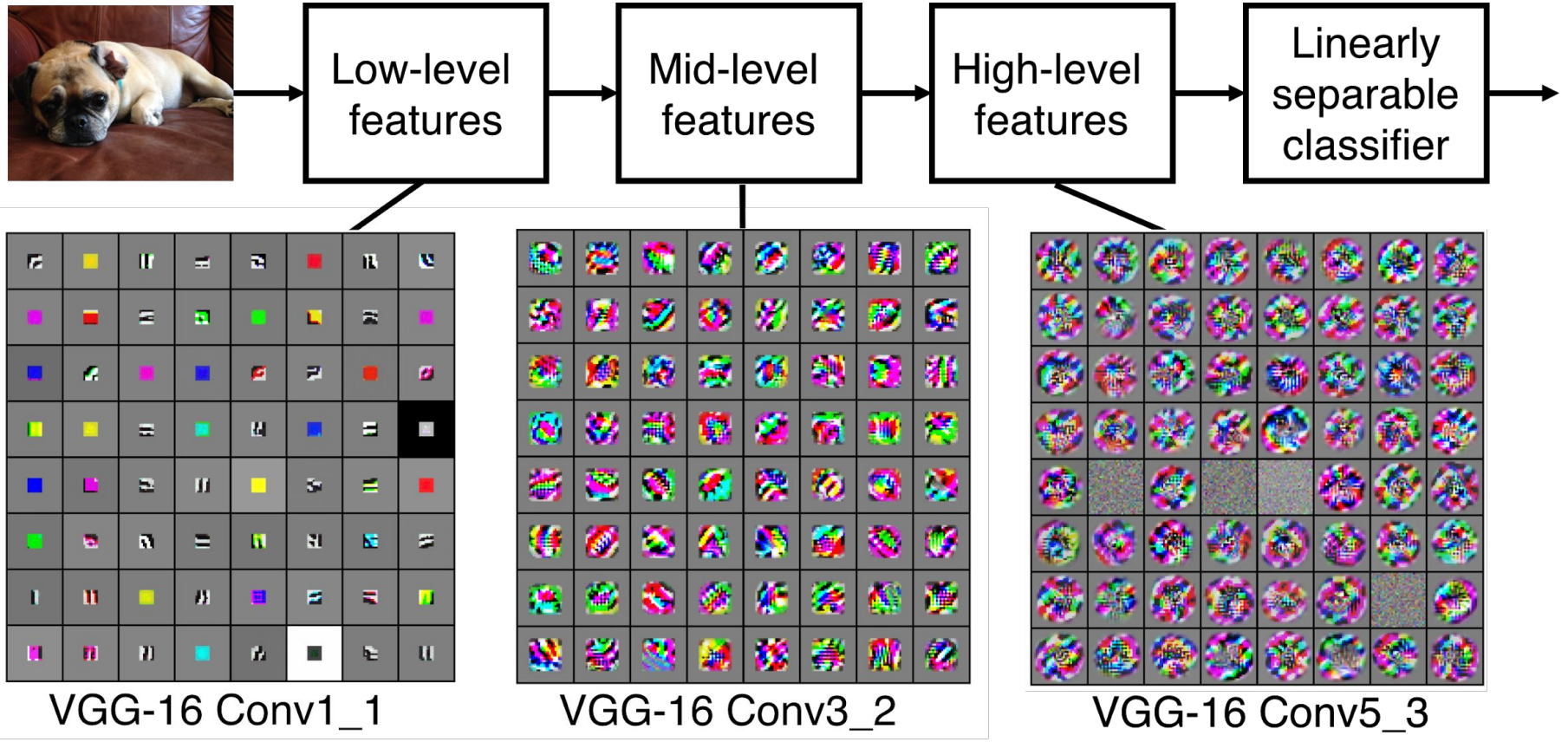
Preview: ConvNet is a sequence of Convolution Layers, interspersed with activation functions



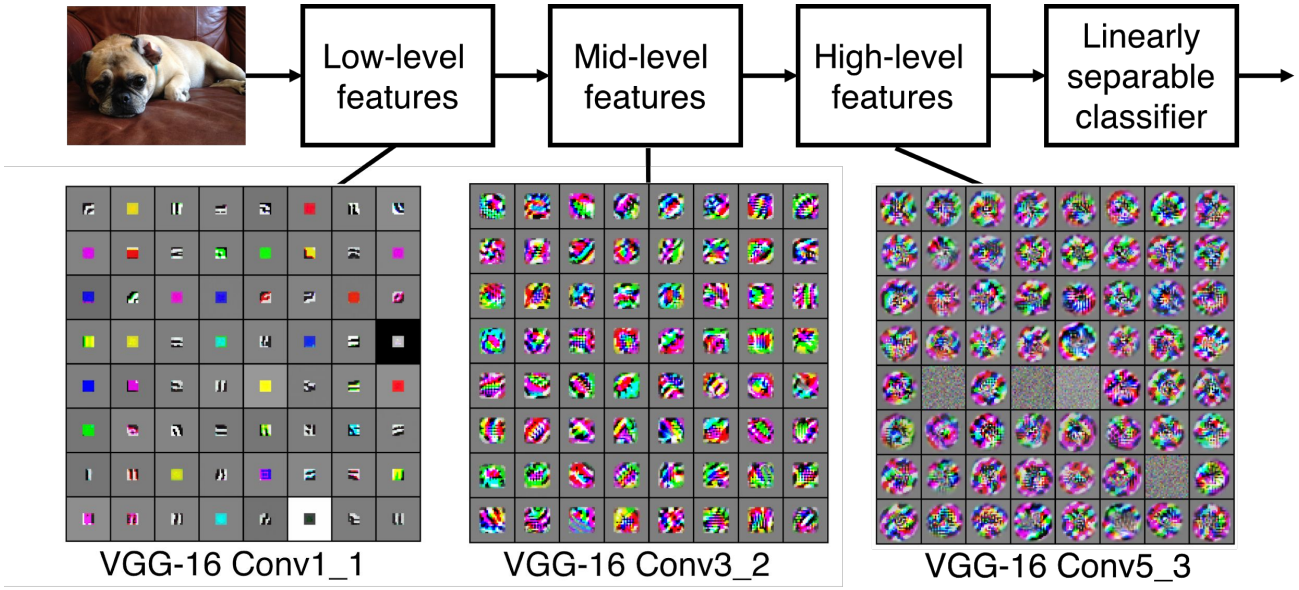
Preview

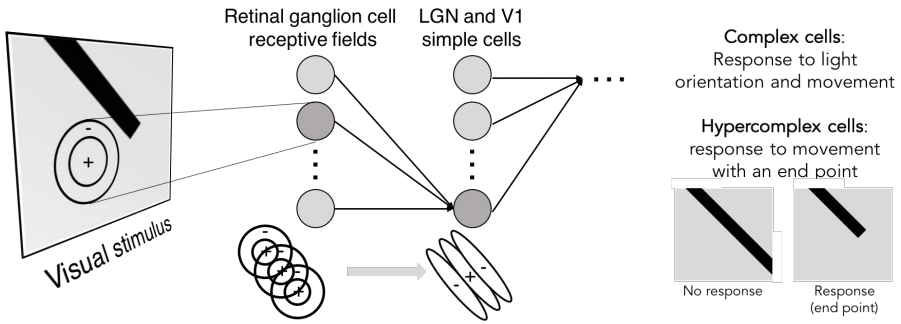
[Zeiler and Fergus 2013]

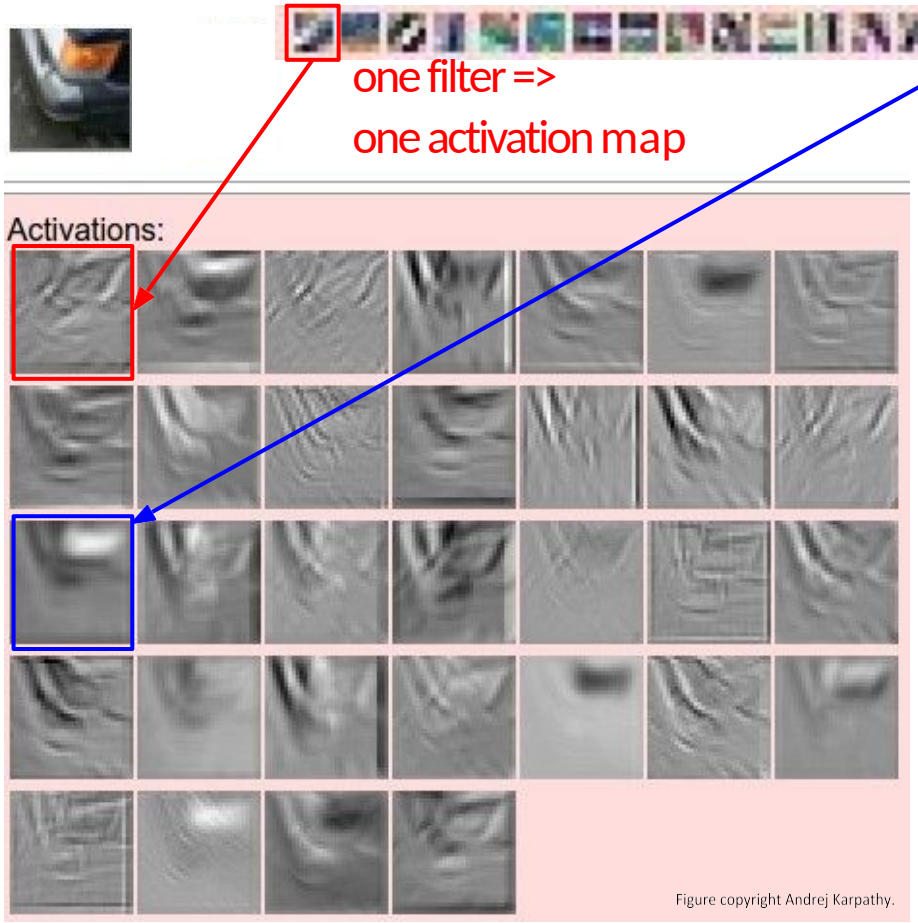
Visualization of VGG-16 by Lane McIntosh. VGG-16 architecture from [Simonyan and Zisserman 2014].



Preview


Low-level features
Mid-level features
High-level features
Linearly separable classifier
VGG-16 Conv1_1
VGG-16 Conv3_2
VGG-16 Conv5_3


Retinal ganglion cell receptive fields
LGN and V1 simple cells
Visual stimulus
Complex cells:
Response to light orientation and movement
Hypercomplex cells:
response to movement with an end point
No response
Response (end point)



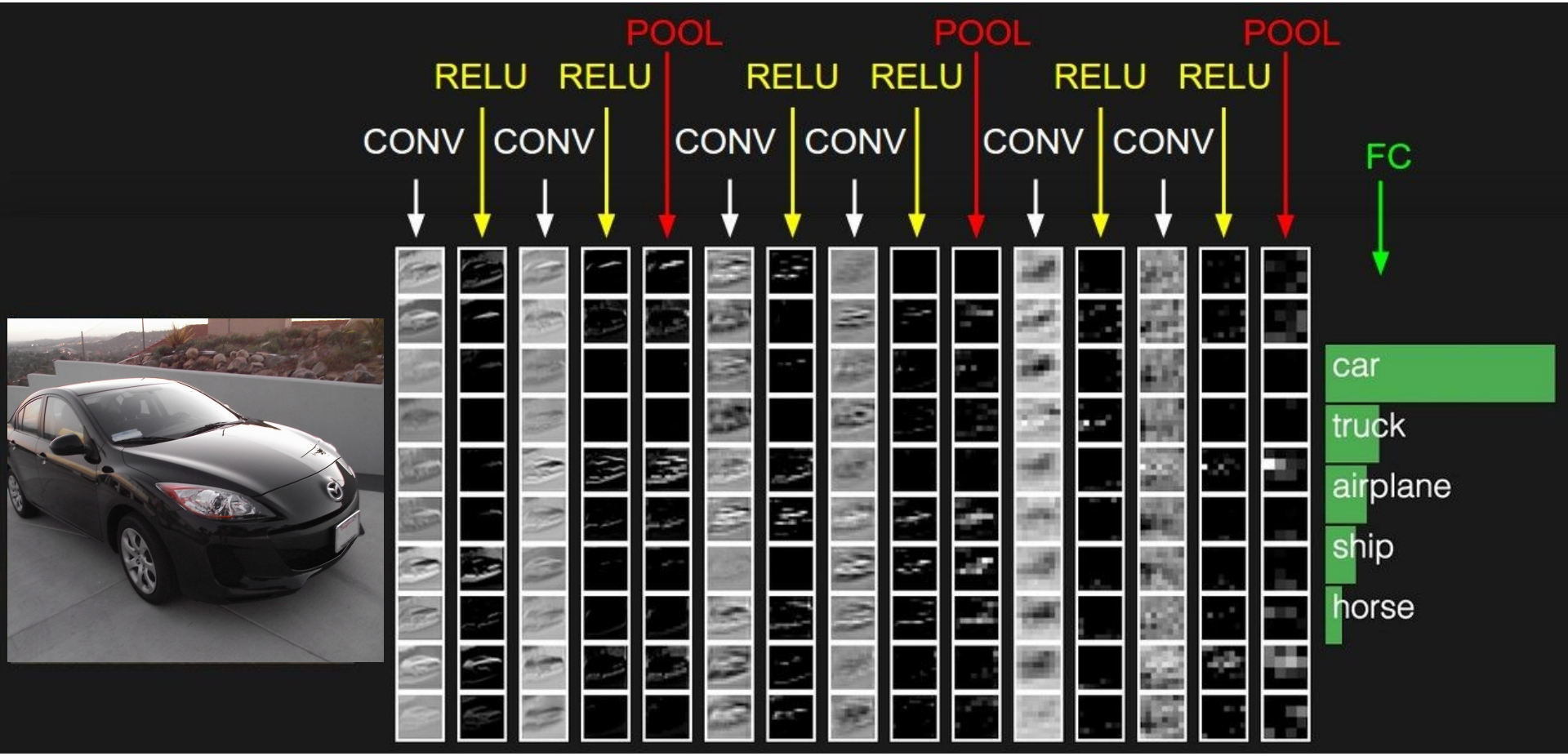
example 5x5 filters
(32 total)

We call the layer convolutional because it is related to convolution of two signals:

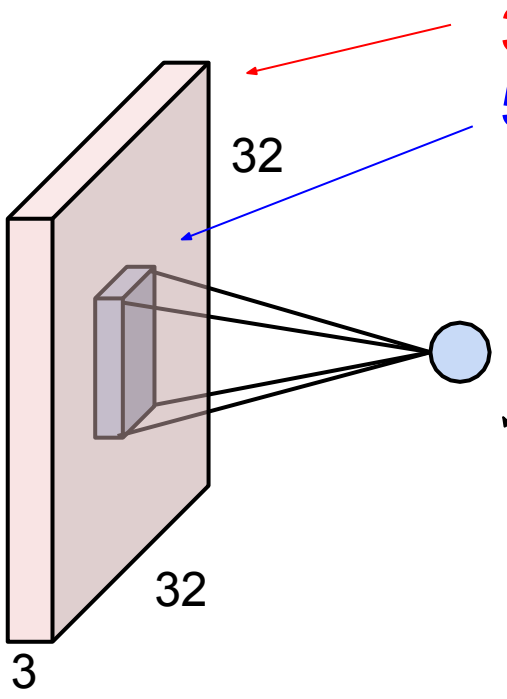
$$f[x,y] * g[x,y] = \sum_{n_1=-\infty}^{\infty} \sum_{n_2=-\infty}^{\infty} f[n_1,n_2] \cdot g[x-n_1, y-n_2]$$

elementwise multiplication and sum of a filter and the signal (image)

Preview:


POOL
POOL
POOL
RELU
RELU
RELU
RELU
RELU
RELU
CONV
CONV
CONV
CONV
CONV
CONV
FC
car
truck
airplane
ship
horse

The brain/neuron view of CONV Layer

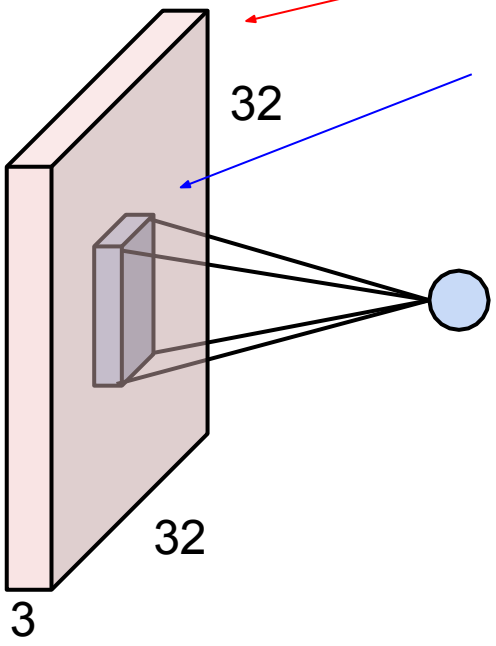


32x32x3 image

5x5x3 filter

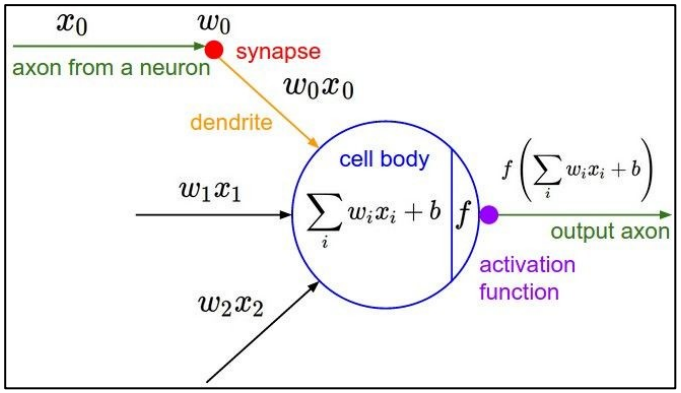
1 number:
the result of taking a dot product between the filter and this part of the image
(i.e. 5*5*3 = 75-dimensional dot product)

The brain/neuron view of CONV Layer



32x32x3 image

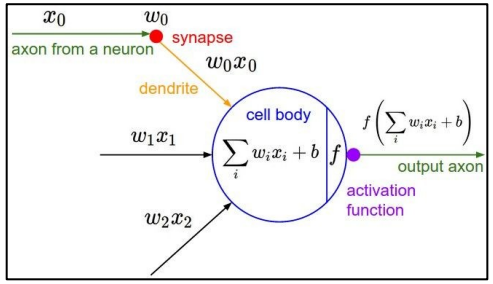
5x5x3 filter

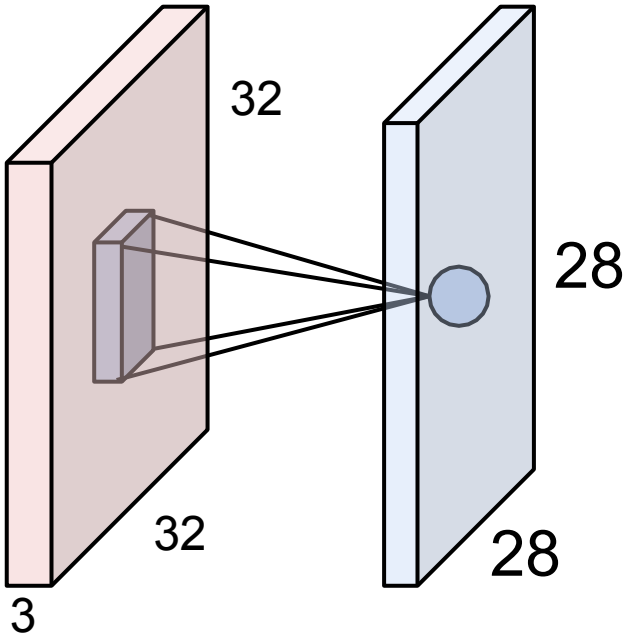


It's just a neuron with local connectivity...

1 number:
the result of taking a dot product between the filter and this part of the image
(i.e. 5*5*3 = 75-dimensional dot product)

The brain/neuron view of CONV Layer



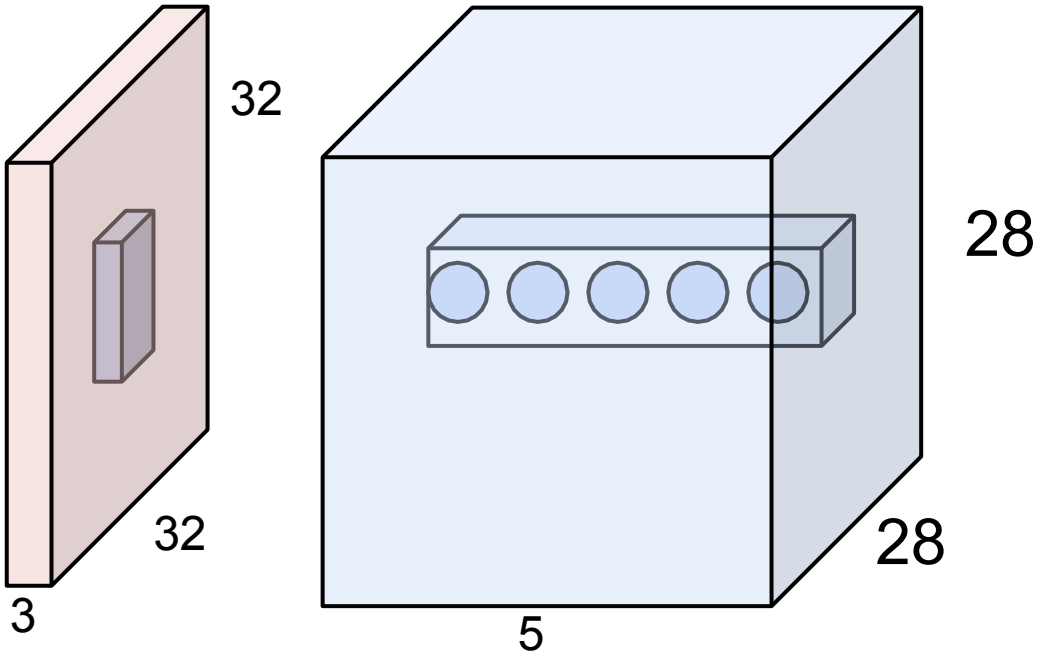


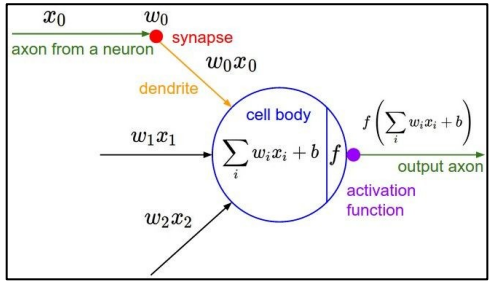
An activation map is a 28x28 sheet of neuron outputs:

1. Each is connected to a small region in the input
2. All of them share parameters

"5x5 filter" -> "5x5 receptive field for each neuron"

The brain/neuron view of CONV Layer

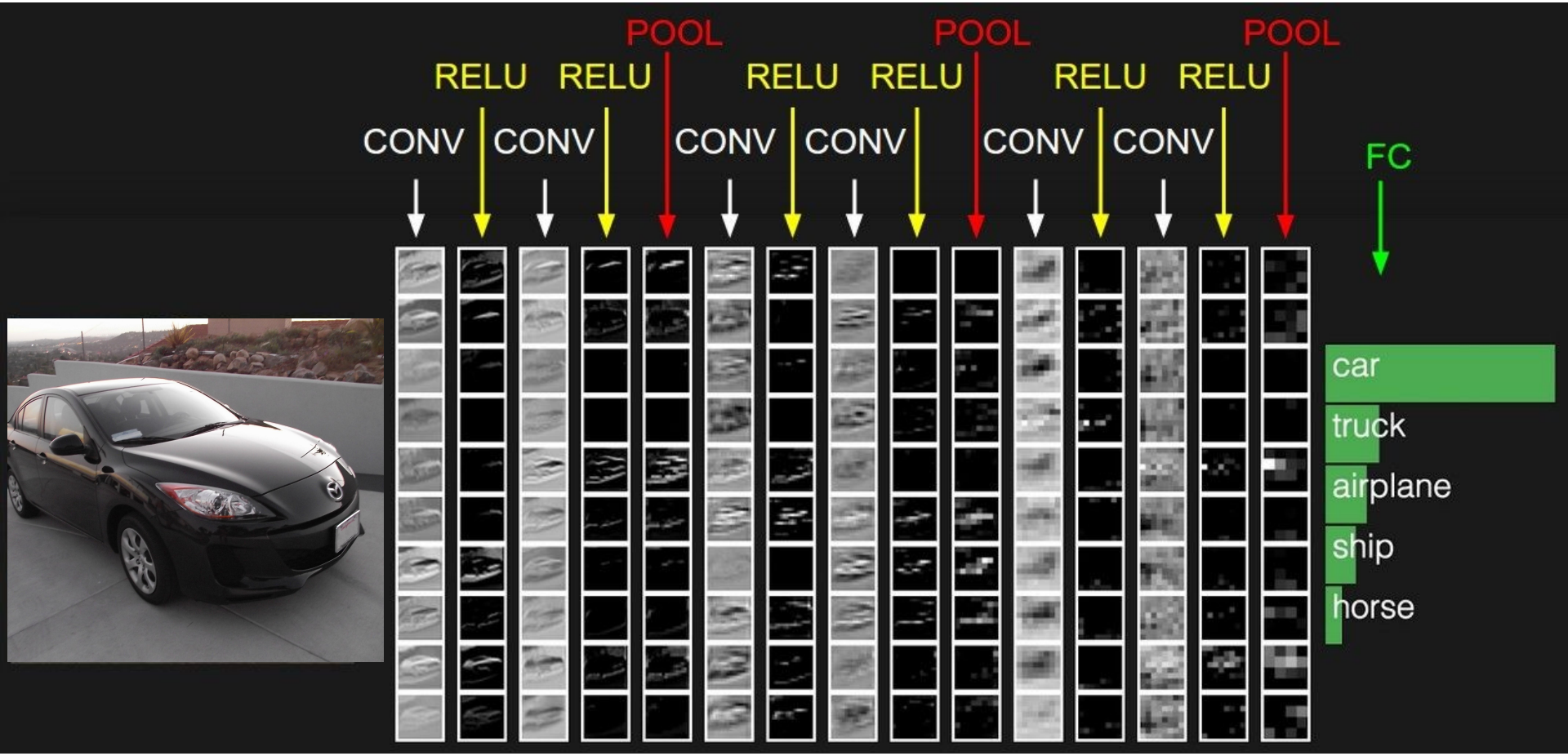




E.g. with 5 filters,
CONV layer consists of neurons arranged in a 3D grid (28x28x5)

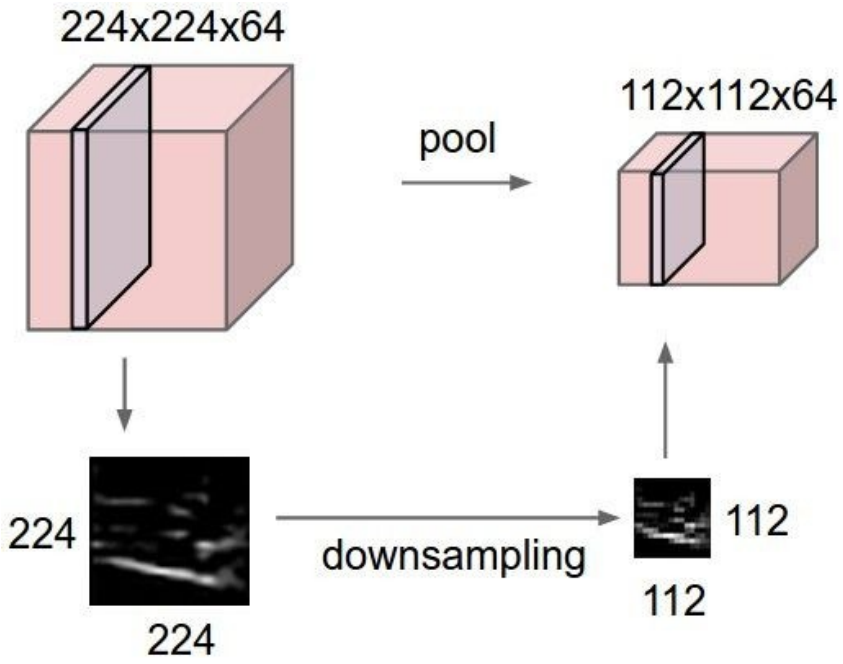
There will be 5 different neurons all looking at the same region in the input volume

two more layers to go: POOL/FC

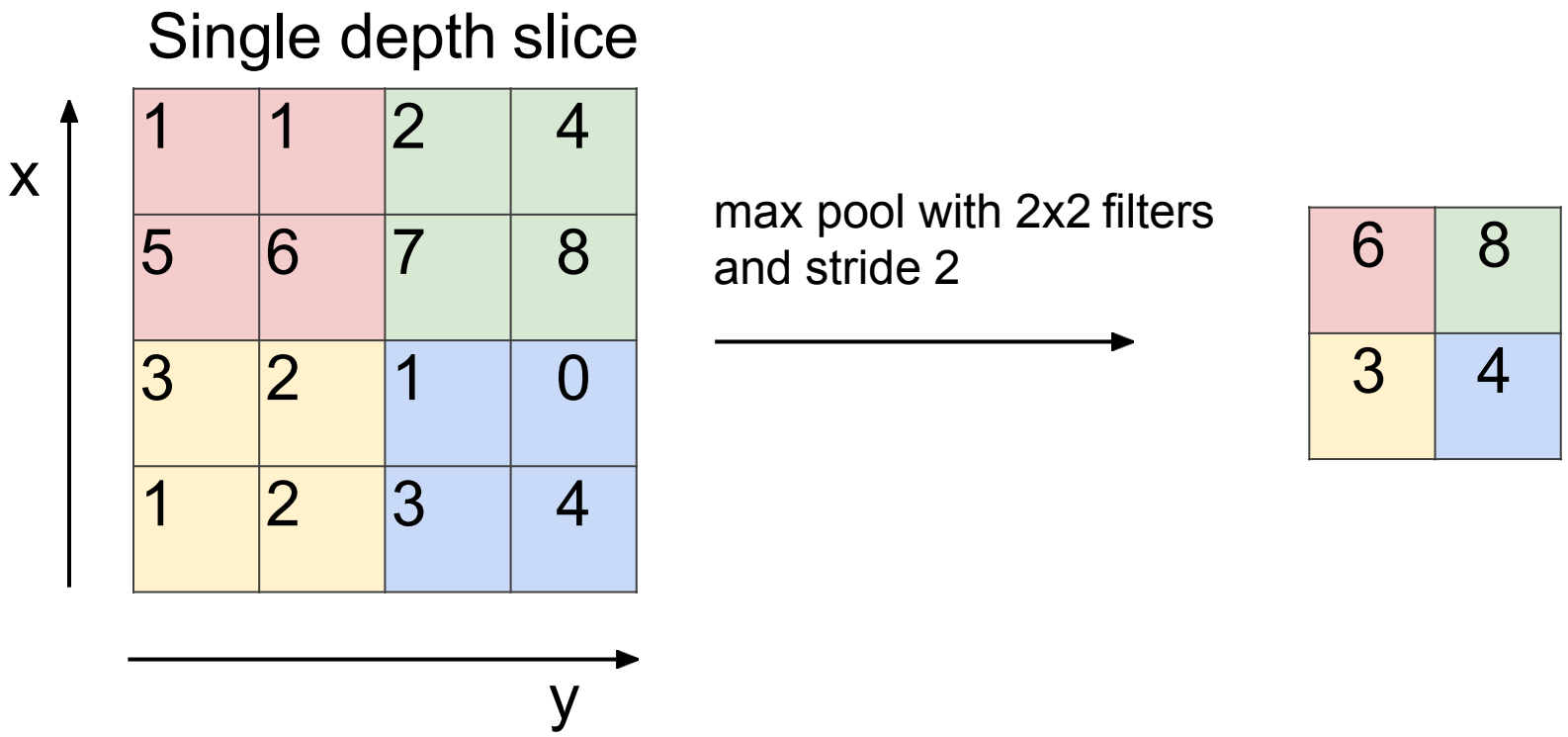


Pooling layer

- makes the representations smaller and more manageable
- operates over each activation map independently:

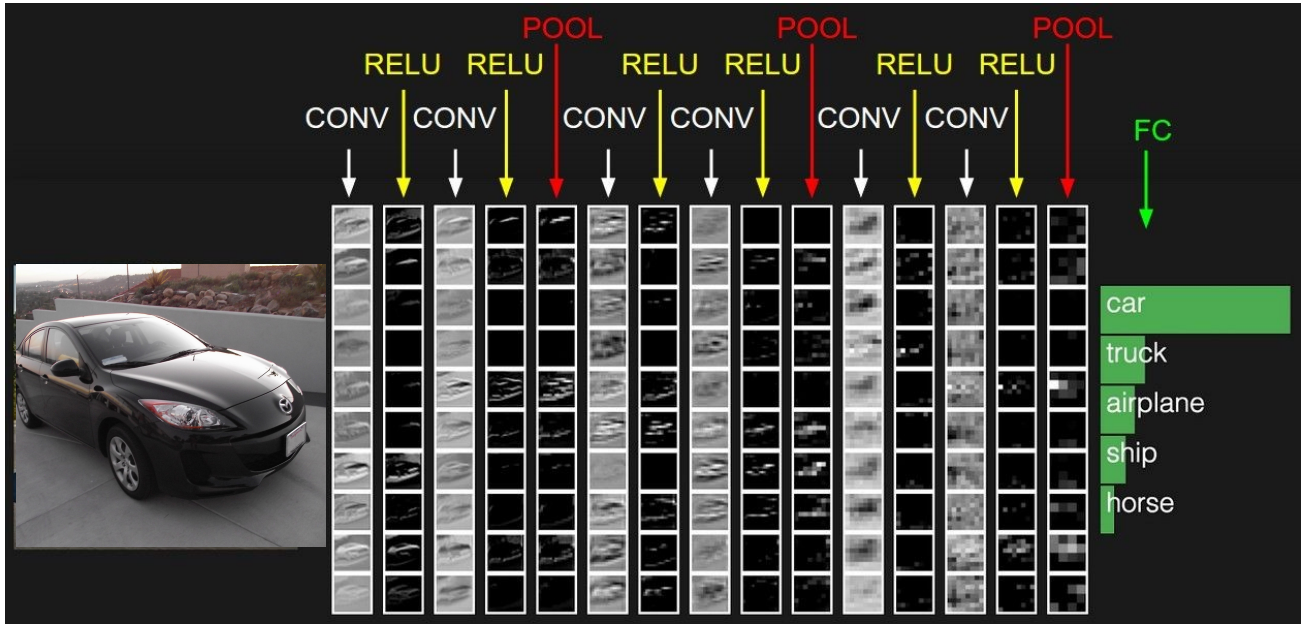


Max Pooling


Single depth slice
x
y
1 1 2 4
5 6 7 8
3 2 1 0
1 2 3 4
max pool with 2x2 filters
and stride 2
6 8
3 4

Fully Connected Layer (FC layer)

- Contains neurons that connect to the entire input volume, as in ordinary Neural Networks



Summary

- ConvNets stack CONV,POOL,FC layers
- Trend towards smaller filters and deeper architectures
- Trend towards getting rid of POOL/FC layers (just CONV)
- Typical architectures look like
  [(CONV-RELU)*N-POOL?]*M-(FC-RELU)*K,SOFTMAX
  where N is usually up to ~5, M is large, 0 <= K <= 2.
  - but recent advances such as ResNet/GoogLeNet challenge this paradigm