Lecture 11: Camera Models

Credit: CS231a, Stanford, Silvio Savarese
Agenda

• Pinhole cameras
• Cameras & lenses
• The geometry of pinhole cameras
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Pinhole camera

\[ f = \text{focal length} \]
\[ o = \text{aperture} = \text{pinhole} = \text{center of the camera} \]
Pinhole camera

\[ P = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \rightarrow P' = \begin{bmatrix} x' \\ y' \end{bmatrix} \]

\[
\begin{align*}
x' &= f \frac{x}{Z} \\
y' &= f \frac{y}{Z}
\end{align*}
\]

[Eq. 1]

Derived using similar triangles
Pinhole camera

\[ P = [x, z] \]

\[ P' = [x', f] \]

\[ \frac{X'}{f} = \frac{x}{Z} \]

[Eq. 2]
Pinhole camera

Is the size of the aperture important?
Shrinking aperture size

- What happens if the aperture is too small?
  - Less light passes through

Adding lenses!
Agenda

• Pinhole cameras
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• The geometry of pinhole cameras
Cameras & Lenses

- A lens focuses light onto the film
Cameras & Lenses

• A lens focuses light onto the film
  – All rays parallel to the optical (or principal) axis converge to one point (the *focal point*) on a plane located at the *focal length* $f$ from the center of the lens.
  – Rays passing through the center are not deviated
Issues with lenses: Radial Distortion

- Deviations are most noticeable for rays that pass through the edge of the lens.

No distortion

Pin cushion

Barrel (fisheye lens)

Image magnification decreases with distance from the optical axis.
Agenda

• Pinhole cameras
• Cameras & lenses
• The geometry of pinhole cameras
  – Intrinsic
  – Extrinsic
Pinhole camera

\[ \mathbf{P} = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad \rightarrow \quad \mathbf{P}' = \begin{bmatrix} x' \\ y' \end{bmatrix} \]

\[ \begin{align*}
    x' &= f \frac{x}{z} \\
    y' &= f \frac{y}{z}
\end{align*} \]

\[ \mathbb{R}^3 \xrightarrow{E} \mathbb{R}^2 \]

\( f = \text{focal length} \)

\( o = \text{center of the camera} \)
From retina plane to images
Coordinate systems

1. Offset

\[ (x, y, z) \rightarrow (f \frac{x}{Z} + c_x, f \frac{y}{Z} + c_y) \]

[Eq. 5]
Converting to pixels

1. Off set
2. From metric to pixels

\[(x, y, z) \rightarrow \left( f \frac{kx}{z} + c_x, f \frac{ly}{z} + c_y \right) \]

[Eq. 6]

Units: 
- \( k, l \): pixel/m
- \( f \): m
- \( \alpha, \beta \): pixel

Non-square pixels
Is this projective transformation linear?

\[ P = (x, y, z) \rightarrow P' = (\alpha \frac{x}{z} + c_x, \beta \frac{y}{z} + c_y) \]  

[Eq. 7]

- Is this a linear transformation?
  No — division by \( z \) is nonlinear

- Can we express it in a matrix form?
Homogeneous coordinates

\[ E \rightarrow H \]

\[
(x, y) \Rightarrow \begin{bmatrix}
  x \\
  y \\
  1
\end{bmatrix}
\]

homogeneous image coordinates

\[
(x, y, z) \Rightarrow \begin{bmatrix}
  x \\
  y \\
  z \\
  1
\end{bmatrix}
\]

homogeneous scene coordinates

• Converting back \textit{from} homogeneous coordinates

\[ H \rightarrow E \]

\[
\begin{bmatrix}
  x \\
  y \\
  w
\end{bmatrix} \Rightarrow (x/w, y/w)
\]

\[
\begin{bmatrix}
  x \\
  y \\
  z \\
  w
\end{bmatrix} \Rightarrow (x/w, y/w, z/w)
\]
Projective transformation in the homogenous coordinate system

\[ P_h' = \begin{bmatrix} \alpha x + c_x z \\ \beta y + c_y z \\ z \end{bmatrix} = \begin{bmatrix} \alpha & 0 & c_x & 0 \\ 0 & \beta & c_y & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} \]

\[ P_h' \rightarrow P' = \left( \frac{x}{z}, \frac{y}{z} \right) \]

Homogenous Euclidian

\[ M = \begin{bmatrix} \alpha & 0 & c_x & 0 \\ 0 & \beta & c_y & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \]

[Eq.8]
Camera Skewness

\[
P' = \begin{bmatrix}
\alpha & -\alpha \cot \theta & c_x & 0 \\
0 & \frac{\beta}{\sin \theta} & c_y & 0 \\
0 & 0 & 1 & 0 \\
0 & 1 & 0 & 1
\end{bmatrix} \begin{bmatrix}
x \\
y \\
z \\
1
\end{bmatrix}
\]

\( C = [c_x, c_y] \)

How many degree does K have? 5 degrees of freedom!
World reference system

In 4D homogeneous coordinates: 

\[ P = \begin{bmatrix} R & T \\ 0 & 1 \end{bmatrix}_{4 \times 4} \begin{bmatrix} x_w \\ y_w \\ z_w \\ 1 \end{bmatrix} \]

\[ P' = K \begin{bmatrix} I & 0 \\ \end{bmatrix} P = K \begin{bmatrix} I & 0 \\ \end{bmatrix} \begin{bmatrix} R & T \\ 0 & 1 \end{bmatrix}_{4 \times 4} \begin{bmatrix} x_w \\ y_w \\ z_w \\ 1 \end{bmatrix} \]

Intrinsic parameters: \( K \)

Extrinsic parameters: \( [R \ T] \)

[Eq.11]
The projective transformation

\[ P'_{3 \times 1} = M_{3 \times 4} \quad P_w = K_{3 \times 3} \begin{bmatrix} R & T \end{bmatrix}_{3 \times 4} \quad P_{w4 \times 1} \]

How many degrees of freedom?

\[ 5 + 3 + 3 = 11! \]
Properties of projective transformations

• Points project to points
• line project to lines, rays or degenerate into points
• Distant objects look smaller
Properties of Projection

- Angles are not preserved
- Parallel lines meet (except for horizontal lines)

Parallel lines in the world intersect in the image at a “vanishing point”
Horizon line (vanishing line)

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