CSE 152 Background Check

This quiz contains 180 points. Please write down the question numbers and answer questions on the blank paper provided. This quiz should take less than 45 minutes. You do not need to show intermediate steps unless you are asked to “prove” or “explain”.

1 Python [5pt]

1. Have you used Python before? Have you used NumPy before? [0pt]
2. Which programming language do you use the most? [0pt]
3. Write a Python snippet that prints numbers from 1 to 100. If you have not coded in Python, write in any language. [5pt]

2 Linear Algebra [15pt]

Consider the following vectors,

\[ \mathbf{v}_1 = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}, \mathbf{v}_2 = \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix} \]

1. What is the dot product \( \mathbf{v}_1 \cdot \mathbf{v}_2 \)? [5pt]
2. What is the cross product \( \mathbf{v}_1 \times \mathbf{v}_2 \)? [10pt]

3 Linear Algebra, matrices [55pt]

Consider the following matrices.

\[ A = \begin{bmatrix} 1 & 1 \\ 2 & 4 \end{bmatrix}, B = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix} \]

1. What is the transpose of matrix \( A \)? [5pt]
2. What is the rank of matrix \( A \)? [5pt]
3. What is the determinant of matrix \( A \)? [5pt]
4. What is the rank of matrix \( B \)? [5pt]
5. Is matrix \( B \) invertible? explain. [5pt]
6. What is the null space of matrix \( B \)? [5pt]
7. If \( R \) is a \( 3 \times 3 \) rotation matrix. What is \( R^T R \)? [5pt]
8. If \( R \) is a \( 3 \times 3 \) rotation matrix. What is the determinant of \( R \)? [5pt]
9. If the linear system \( Bx = b \) can be solved, what is the form of vector \( b \)? [5pt]
10. Definition: a transformation $T$ is linear if and only if given any vectors $\mathbf{x}_1, \mathbf{x}_2$ and scalar $a$, $T(a\mathbf{x}_1 + \mathbf{x}_2) = aT(\mathbf{x}_1) + T(\mathbf{x}_2)$. Now, given linear transformations $T_1$ and $T_2$, prove $T_1 \circ T_2$ is linear. 

hint. $T_1 \circ T_2(\mathbf{x}) = T_1(T_2(\mathbf{x}))$

4 Linear Algebra, calculation. 

Consider the following matrix,

$$C = \begin{bmatrix} 1 & 6 \\ 1 & 2 \end{bmatrix}$$

1. Solve for $x_1, x_2$, where

$$\begin{bmatrix} 1 & 6 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 5 \\ 1 \end{bmatrix}$$

[10pt]

2. What are the eigenvalues and eigenvectors of matrix $C$? 

[20pt]

5 Calculus

Consider the following functions

$$f(x) = x^2, \quad g(x) = \sin(x), \quad h(x, y) = x^2y$$

1. What is the derivative of $f(x)$? [5pt]

2. Sketch a graph for $g(x)$. [5pt]

3. Sketch a graph for the derivative of $g(x)$. [5pt]

4. What is the partial derivative of $h$ with respect to $x$, $\frac{\partial}{\partial x} h(x, y)$? [5pt]

5. What is the mixed second order partial derivative $\frac{\partial^2}{\partial x \partial y} h(x, y)$? [5pt]

6. What is the gradient of $h(x, y)$, $\nabla h(x, y)$? [5pt]

7. State the chain rule of differentiation. [5pt]

6 Probability

1. Given $P(A) = 0.7, P(B) = 0.2, P(A \cap B) = 0.1$, what is $P(A \cup B)$? [5pt]

2. Given the above, what is $P(A|B)$? [5pt]

3. Express $P(A|B)$ in terms of $P(A), P(B)$ and $P(B|A)$. [10pt]

4. (hard) Given random variable $X$, prove or disprove $\mathbb{E}[X^2] \leq \mathbb{E}[X^2]$. [10pt]

5. (hard) Given random variables $A, B, C$. $A$ and $B$ are independent, $B$ and $C$ are independent. Are $A$ and $C$ independent? If not, give an example. [10pt]